

GENERAL DESIGN OF A SOLAR POWER SYSTEM USING THE PRINCIPLE  
OF ENERGY AVAILABILITY

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GENERAL DESIGN OF A SOLAR POWER SYSTEM USING THE PRINCIPLE  
OF ENERGY AVAILABILITY

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## NOMENCLATURE

A	area
B	availability
E	energy
H	enthalpy, $H = E + P V$
N	quality of matter
P	absolute pressure
Q	heat
S	entropy
T	temperature
U	heat transfer coefficient
V	volume
W	work
$W_L$	lost work
$\wedge$	availability
$\mathcal{A}$	available energy
$\mathcal{E}$	essergy
$\mu_{co}$	Gibbs potential
$\theta$	irreversibility
$\eta$	efficiency
$\gamma$	intercept factor
$\alpha$	absorptivity
$\epsilon$	emmissivity

## NOMENCLATURE (Continued)

$\tau$	transmissivity
$\sigma$	Stefan-Boltzmann constant
$b$	availability per unit mass
$e$	effectiveness
$g$	gravity
$h$	enthalpy per unit mass
$m$	mass
$q$	heat per unit mass
$r$	reflectivity
$s$	entropy per unit mass
$u$	internal energy per unit mass
$v$	volume per unit mass

## Subscripts:

$a$	aperture
$c$	collector
$f$	working fluid
$h$	high
$i$	initial condition
$l$	low
$p$	product
$r$	reservoir
$t$	turbine
$\sigma$	open system
$( )_s$	constant entropy

## ABSTRACT

The purpose of this research was to design in general a solar power system using the latest solar technology. Energy availability methods were used as a guide in the initial design decisions. It was found that the effectiveness of a power cycle is virtually constant with respect to the collector temperature--a principle which may be called the principle of constant effectiveness. For idealized situations where the capital cost of a power plant is taken to be independent of the boiler steam temperature, this principle of constant effectiveness dictates an optimum boiler temperature of  $4452.6^{\circ}\text{R}$ . Thus, for any actual plant, it is strictly the dependence of material costs on temperature which dictates lower optimum boiler temperatures. In other words, there is nothing in the laws of physics concerning collector efficiencies which points toward a lower optimum temperature than  $4452.6^{\circ}\text{R}$ .

In view of this conclusion from energy availability methods, it appeared that the best way to proceed in utilizing the latest technology in the design of a solar power system was to utilize a high temperature power cycle which is widely used and developed, thus allowing us to reach the highest temperature used in today's conventional power plants. We thereby selected the Rankine power cycle for our study. Appropriate performance specifications were found to be

- a) A solar collector (parabola of revolution mirror) of high effectiveness (60.47%) at reasonable cost ( $\$7.5/\text{ft}^2$  or less).



- b) The diameter of the solar collector is five feet.
- c) The receiver type is of the cavity type (called a solar well), the aperture diameter being about 0.6 inch.
- d) A tracking system that keeps the sun's rays focused within the aperture. To aid in this, we used a rotating aperture coupled with an automatic control system.

A solar power plant which meets these specifications was then designed in general. This power plant needs  $3.257 \times 10^7$  square feet of collector surface for a 1000 megawatt daytime output and  $8.3 \times 10^6$  feet of insulated pipe for delivery of the working fluid. The rough capital cost estimate for the entire solar power plant is  $4.35725 \times 10^8$  dollars. This figure leads to a power cost of 2.1¢/cw-hr--or over twice the cost of power plant from a conventional steam power plant. Most of this cost is due to the cost of the collector system ( $3.3575 \times 10^8$  dollars).



## CHAPTER I

### INTRODUCTION

#### Problem Statement

Man needs energy to live, to have a better life. As agricultural societies change into industrial societies, the rate of energy consumption is greatly increased, since industrial societies depend on the use of power. The more advanced the industrial society, the more energy will be consumed, resulting in growth and a higher quality of their life.

In 1965, Mr. L. P. Gaucher<sup>1</sup> presented the graphs which are shown in Figures 1 and 2 at The Solar Energy Society meeting in Phoenix, Arizona. Those graphs show us that 30-40 years from now a new large source of energy will be needed to supplement nuclear energy and the world dwindling supply of fossil-fuels. It was suggested then that this new large source of energy might be solar energy.

In Figure 3, the various sources of energy arranged in order, in which they were developed chronologically and this graph shows us very clearly that the United States has developed a new source of energy every 30-40 years.

Everyone is aware of the technical and economic limitations of solar energy--its low density and terrestrial variability. Despite these drawbacks, it provides the Earth with 178 billions of megawatts per year.<sup>3</sup> It is obvious that solar energy has the advantage of being available everywhere with no cost of supply or distribution.

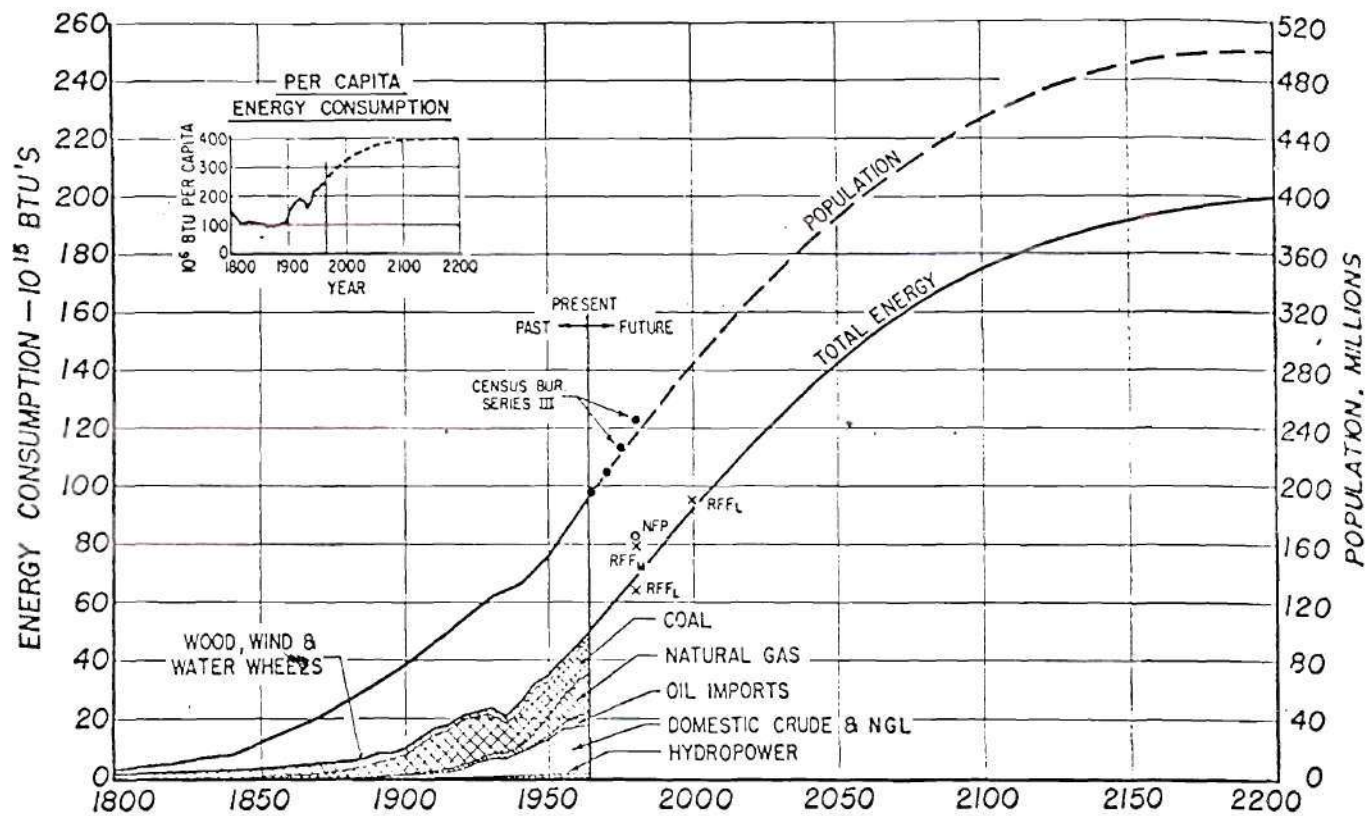


Figure 1. Energy Consumption in the United States, Past, Present, and Future

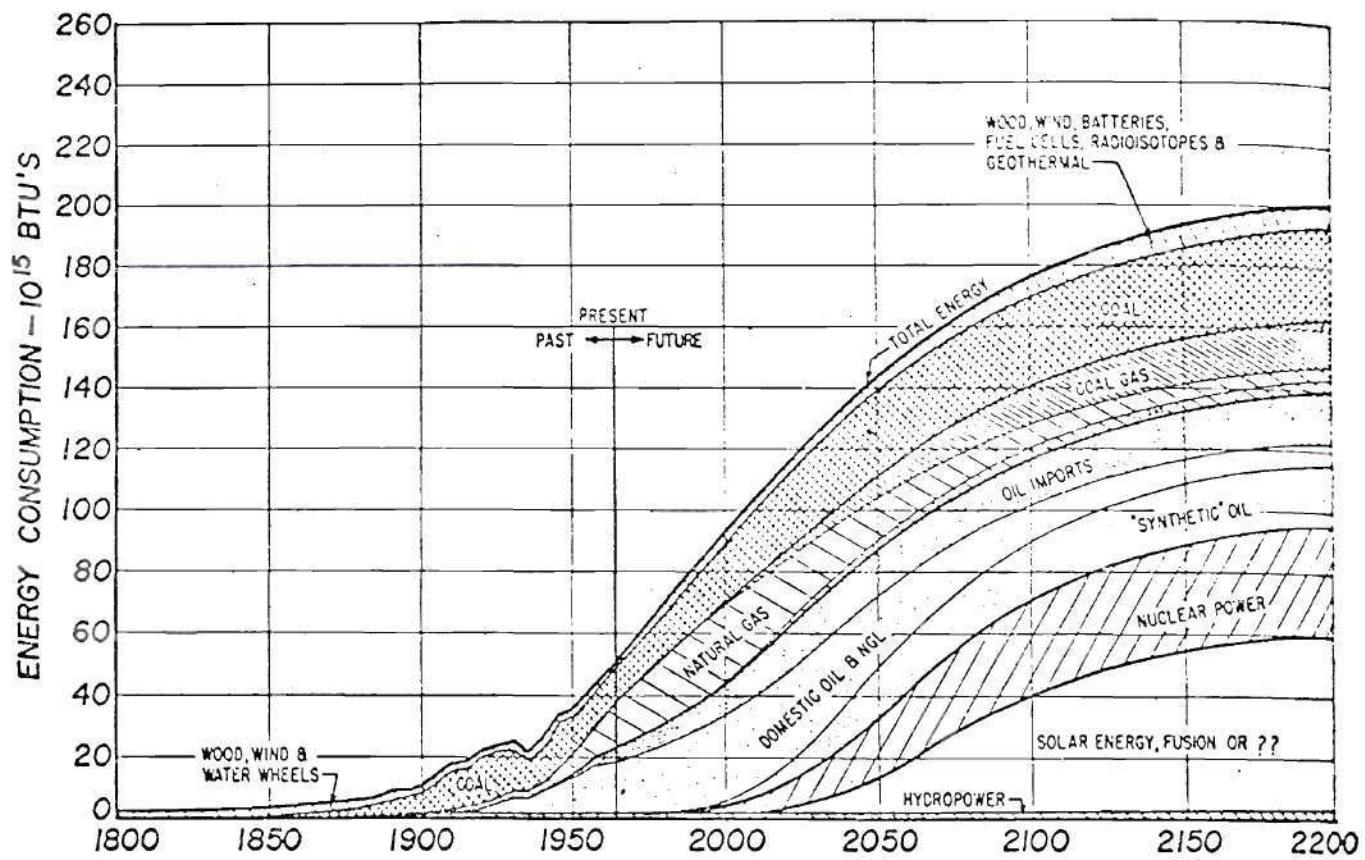


Figure 2. Energy Consumption in the United States, with the Future Given in Detail

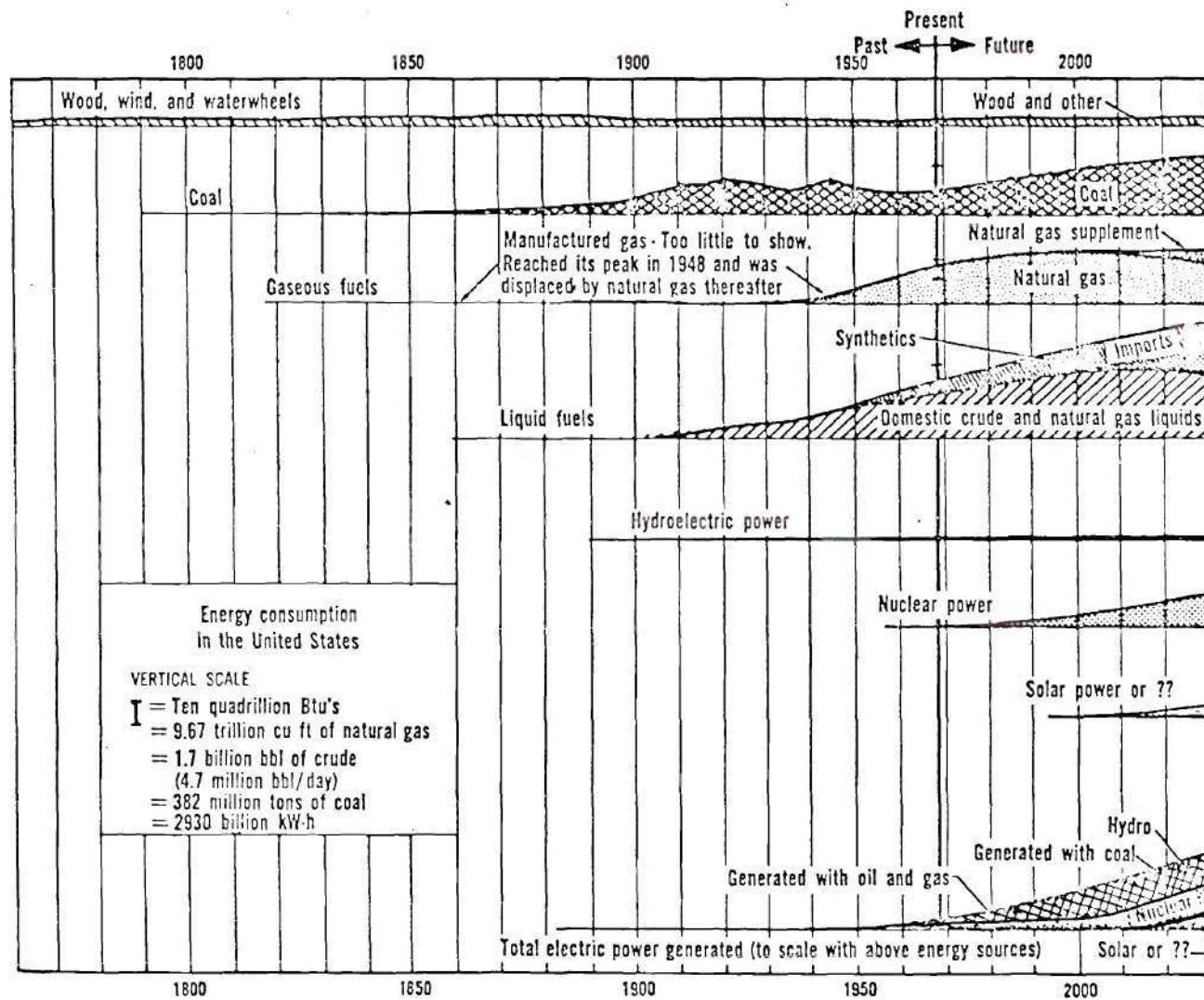


Figure 3. Chronological Development of Energy Sources



The factors which adversely affect the practicality and cost of solar energy are high variability due to regular hourly daily and seasonal changes and irregular cloud cover variations, and its low relative density. Table 1 summarizes the average availabilities of solar energy.

The variability of solar energy can be compensated for by means of standby power, storage schemes, etc. In any case, it is reasonable to determine a practical way to incorporate solar energy into a power grid so that the combination can meet any reasonable design standards as to load reliability.

There is increasing concern about air pollution, water pollution, and more recently thermal pollution. The costs of mitigating these effects are expected to be quite large. Faced with such significant investment for power generating plants, we should at least explore alternative approaches before the world-wide demand for energy reaches a point where the adverse effects of the living environment become intolerable. We should consider solar energy as our major future resource and evaluate the harnessing of it. The current "energy crisis" makes the solar energy utilization effort all the more urgent. Energy from the sun has a promising future. Admittedly such systems now cost more to build than conventional plants and this cost must be reduced.

The need for a new source of energy for generating electric power has been well established. We are in a position to make such an evaluation based on technology and on the attractiveness of solar power. The purpose of this research is to outline the design of a solar power system which will reflect the latest solar technology. In this thesis the main thermal characteristics of the design are set forth.

Table 1. Average Availability of Solar Energy<sup>3</sup>

Availability Factor	Average on Earth	In Synchronous Orbit	Average Ratio
Solar insolation	0.11 W/cm <sup>2</sup>	0.14 W/cm <sup>2</sup>	4/5
Percentage of clear skies	50%	100%	1/2
Cosine of angle incidence	0.5	1.0	1/2
Usual duration of solar irradiation	8	24	1/3
Product			1/5

NOTE: Column 2 gives average solar insolation on earth.

Column 3 gives average solar insolation on synchronous orbits.

Column 4 gives the average ratio of earth and synchronous orbits.

## CHAPTER II

### LITERATURE SURVEY

Technically, solar energy systems have been shown to be feasible in that it is possible to design and construct systems that will convert solar energy for various purposes such as electricity and refrigeration. Many of these systems are relatively simple from an engineering viewpoint and do not require unusual manufacturing.

There are many hundreds of solar energy utilization schemes presented in the literature. What follows here is a survey of several of them, selected on the basis of successful application of a research effort.

#### Solar Collectors

In overcoming the disadvantage of low solar intensity, various types of collectors are used. "Collector type" provides a convenient classification for a device because of collector influence on a system's thermodynamic characteristics and cost. Collectors can be considered either of the intensive or extensive variety. Intensive collectors are characterized by an optical concentration system, high temperature, high energy fluxes, and higher unit cost. With the optical system and tracking gear they cost more than \$3/sq ft.<sup>4,5</sup> Extensive collectors are characterized by large collectors' areas, low temperature, low energy flux, and lower unit cost, less than \$1.00/dq ft.<sup>5,6,7</sup>

Intensive collectors use some optical concentrator (mirror or lenses)

such as paraboloidal, spherical, conical, parabolotoridal, parabolocylindrical, and other types of concentrators. A patent search yields very little information on intensive collectors, as discussed in Appendix C, so the following material is from the general literature.

Various types of reflectors have been proposed.<sup>8</sup> The most important of these are semirigid reflectors of the umbrella type<sup>8,9</sup>; the component elements are high-grade paraboloidal mirrors in sizes up to at least 10 or 12 square inches, which reflect the sun's rays to the target. This reflectance is maintained at about 80%. In addition to paraboloid mirrors, Fresnel reflectors also appear to have desirable characteristics.<sup>10</sup> This type consists essentially of a flat plate with a number of concentric rings, each of which is cut at a different angle so that all incident radiation is directed toward a common focal point.

The conical concentrators are analyzed in reference 11 for two types of receivers--cylindrical and conical. At the optimal apex angle of  $90^\circ$  the conical reflector creates a greater concentration than the parabolocylindrical type. The energy distribution over a conical receiver is uniform.

Other types use lens concentrators. Making large lens-type concentrators is much more difficult and expensive than making reflector-type concentrators of the same size. In solar energy systems, no record has been found of the use of lens concentrators.

The extensive collectors are of the stationary flat-plate type. They are cheaper than intensive collectors and utilize both diffuse and direct radiation. In the periodical, "Solar Energy," there are many papers discussing this type of collector. The relative advantages (+)



and disadvantages (-) of intensive and extensive type collectors are summarized in Table 2.

### Uses

#### Salt Production

Perhaps the oldest successful application of solar energy is the production of table salt by evaporation of salt containing waters.<sup>12</sup>

#### Solar Water Still

Production of fresh water from sea or brackish water by solar evaporation is a very attractive process.<sup>12-17</sup>

A typical solar still is a large plastic covered, rubber lined, tank about 2 cm deep; sea water is pumped in continuously and high flow rates maintained to prevent salt concentration. For such a still, evaporator area cost per 1,000 gallons is between \$3 and \$4.<sup>16</sup>

#### Solar Water Heater

Solar water heaters are utilized for laundries, swimming pools, and hot-water heating systems.<sup>12,13,18-21</sup> In many countries where the incident solar radiation is greater than  $300 \text{ cal/cm}^2$  per day, solar production of plentiful hot water for domestic use is economically competitive with other processes. The water is heated from the side of the open surface. For preheating water to low temperatures, shallow basins are used. The temperature of the open water can be increased by covering the surface with oil from drip pans which float on its surface. This decreases the expenditure of heat on the evaporation of water. A typical domestic installation is roof mounted and consists of a storage tank pump topped shallow basins through which the water is pumped.

Table 2. The Relative Advantages and Disadvantages of Intensive and Extensive Type Collectors<sup>7</sup>

	Extensive Type	Intensive Type
Operation	Stationary	Moving
Efficiency	Low (-)	High (+)
Heat Loss	High (+)	Low (-)
Temperature	Low (-)	High (+)
Cooling	Difficult (-)	Easy (+)
Solar	Direct and Diffuse (+)	Direct (-)
Radiation	Sun High in Sky (-)	All Day (+)
Capital Cost	Low (+)	High (-) ?
Labor Cost	Low (+)	High (-) ?

NOTE: Advantage (+); Disadvantage (-)

### Solar Cooker

The use of solar-powered devices for food preparation has long been advocated.<sup>12</sup> A typical device is an aluminized, plastic, parabolic reflector focusing on a simple stand holding the cooking utensil. The cooker is heated with direct sun rays and after reflection from the parabolic reflector.

### Hot Air Engine

The replacement of the chemical or nuclear source of heat engine by solar energy is an intriguing idea, but unfortunately it is not a practical idea for engines larger than one kw.<sup>5,22-26</sup> The difficulty arises from the need for focusing collectors in order to obtain reasonably high temperatures. Collectors of this type have a geometry difficult to fabricate, ship, and assemble. They are limited by the requirement that wind loads be withstood. Tabor<sup>5</sup> discusses mechanical solar power and indicates the economic limit to these machines is about one kw output. Both Farber<sup>24</sup> and Daniels<sup>23</sup> indicate that fractional horsepower, solar engines are feasible as electric generators. A 50-Watt unit has been reported which has a weight of 15.5 lb, costs \$470 each in limited production, has an overall efficiency of 7.5%, and has a size of 16 in. by 17 in. by 26 in. with mounting system and collector folded.<sup>25</sup>

### Solar Heating and Cooling of Buildings

With proper orientation and construction of windows and planning of buildings, systems were proposed for heating rooms with solar energy stored in heat accumulators to prevent rooms from being overheated by solar rays.<sup>5,12,13,27-29</sup> A solar heated home in Cairo, Egypt is reported

in reference 27. Perhaps the most interesting design for a solar heated home is built by H. Thomason in the vicinity of Washington, D. C. Typically, a small home is heated in winter and cooled in summer by water circulating over a roof designed to heat the water at night by radiation (for summer time). This roof top heat exchanger has an area of about 5,000 ft<sup>2</sup> and is fashioned by covering corrugated blackened aluminum with a polyester film, and polyester film with ordinary window glass. A circulation pump pumps water to the peak of the roof. The water runs over the aluminum and under the polyester film to the ground level, then into a heat exchanger. Air is circulated around the heat exchanger and house. The heat exchanger is a 1,600 gallon steel tank filled with water surrounded by 50 tons of gravel. The heat exchanger is heated (or cooled) during the day (or night) and is used to store energy for heating (or cooling) during the day when the sun is not heating the circulating water. The current cost of this system is about \$2,500.

#### Solar Furnace

Archimedes is reputed to have defended Syracuse with a solar furnace. More recently several large solar furnaces have been built in the United States and other countries.<sup>12,30,31</sup> A 35 kw unit operates at Sendai, Japan; a 35 kw unit was in operation at Natick, Massachusetts; and the world's largest is a 1 MW unit at Odeillo, France. These equipments were built as research tools for metal fabrication, high temperature materials, ultrapure alloy production, and high temperature chemistry. The Odeillo installation has the following characteristics: sun rays are collected by 63 heliostats 24.6 ft x 19.7 ft planer reflectors, each individually guided and focused on the main parabolic mirror. The main mirror



is 175 ft x 130 ft and is composed of some 9,500 small mirrors. The focus length is 59 feet. One MW of thermal power is concentrated in an area about 6 inches in diameter, delivering a maximum flux of  $1,600 \text{ watts/cm}^2$  (equilibrium temperature is approximately  $4,100^\circ\text{K}$ ). This unit cost,  $\$2 \times 10^6$  or  $\$2,000/\text{kw}$ , includes the research buildings and equipment.

#### Solar Pond

The National Physical Laboratory of Israel has investigated the concept of "solar pond," a shallow pond stabilized in a non-free convection mode by strong salt gradients.<sup>32-34</sup> It is claimed that solar radiation increases the temperature of the bottom layer of water to a degree that makes recovery of this energy feasible. This water would be decanted from a pool about one meter deep at about  $95^\circ\text{C}$  and flashed to steam to run a more or less conventional steam turbo-generator set. It is estimated that a 25 square mile pond would produce 160 MW at a capital cost of  $\$300/\text{kw}$  as compared to about  $\$200/\text{kw}$  for a conventional plant.

#### Portable Fuels Production

Concern for the depletion of fossil-fuels and environmental effects, mobil prime movers, has prompted several investigations to examine the possible application of solar energy to produce the portable fuels. The most persistent investigation is the electro-decomposition of water by solar produced electricity. Decomposers can be built at about  $\$25/\text{kw}$  and achieve almost 100%  $\text{O}_2$  and  $\text{H}_2$  production/faraday.<sup>32</sup> The oxygen and hydrogen can be recombined in an efficient oxygen-hydrogen fuel cell. Other uses have been proposed for just hydrogen, such as the production of ammonium nitride or methyl-alcohol fuels that can be burned directly.

Several schemes for fuel production have been proposed to take advantage of high temperatures produced by focusing collectors. The decomposition of molten lithium hydride at  $900^{\circ}\text{C}$  is a reversible reaction. The recombination yields electrons harnessable by a fuel cell. High temperature and proper catalyst will produce nitric acid, which can be further processed into ammonium nitrate that is a source of much stored energy.

The most interesting fuel production plans use photochemical reactions. The most promising decomposition is nitrosyl chloride, using light to yield nitric oxide and chloride.<sup>35</sup> A 26% conversion efficiency was claimed. The products can be recombined in a highly efficient fuel cell to produce electricity.

#### Air Conditioning and Refrigeration

Most of such equipment, constructed or proposed, utilized hot-water, water, or ammonium vapor produced in solar boilers or other equipment. Solar powered air conditioning built by Farber and Flanigan is reported in reference 37. It was a 2.4 ton machine with a coefficient of performance of 0.57. About 400 square feet of concentrating collector was used. Solar powered refrigeration systems using ammonia absorption techniques are operating and seem practical for large scale production devices.<sup>37</sup> Other plans are jet cooling devices<sup>36</sup>; ammonia water, ammonia sodium thracyanate<sup>38</sup> absorption devices. A Freon-12 solar cooler is reported by Kakabaev and Davletov.<sup>39</sup> This device supplies air at  $14^{\circ}\text{C}$  lower than ambient with a 20% efficiency. A solar refrigeration system having ammonia-sodium thiocyanate is reported by R. K. Swartman and V. H. Ha.<sup>40</sup>

### Solar Cells

The most widely used solar energy conversion device is the semiconductor solar cell.<sup>5,13,12,26,41-46</sup> The state of the art of solar energy conversion is evident when it is considered that solar cells cost from \$10,000/kw to about \$100,000/kw for manned, space certified units.<sup>13</sup> The reason for these high costs are low conversion efficiency, expensive materials and expensive fabrication of arrays. Research is being conducted to improve the three types of inorganic solar cells. They are the silicon cell, cadmium sulfide cell, and gallium arsenide cell. The silicon cell has obtained a conversion efficiency of 10-12% but suffers from radiation damage. The cadmium sulfide cell has achieved an efficiency of 6%. It is resistant to the radiation damage but sensitive to moisture. The gallium arsenide cell has an efficiency of 9%.<sup>5,12,42</sup>

Presently the basic cost of silicon is about \$100/lbm or \$30/kw.<sup>47</sup> These cells are very small. It is estimated that a continuously produced, vapor-deposited, thin solar cell will be available at about \$1,000/kw in the future.<sup>26,42-44</sup>

The Washington, D. C. area power requirements are  $1.1 \times 10^{10}$  kw-hr/yr, which puts the use of solar cell power into perspective. With a 7% conversion efficiency, 73 one-square-mile generating plants are required. Associated with each plant will be a building of  $400,000 \text{ ft}^3$  containing enough storage batteries to store  $10^6$  kw-hr, providing around-the-clock power and 4-day reserve. The 20 year aggregate cost of each square mile station is  $\$100 \times 10^6$ .<sup>45</sup>



### Integrated Solar Farm

Solar energy converted via photosynthesis appears to be the ultimate energy conversion system.<sup>13,51</sup> One such conversion scheme is operating on the Pacific coast of Mexico. A diesel engine is used to pump sea water into evaporators where the solar energy and the diesel's rejected heat evaporate fresh water. The fresh water is pumped to greenhouses and waters the vegetables. Another project is under way in Abu Dhabi, Arabia.

### Solar Power Stations

Solar radiation is most economically converted into heat, and heat into electrical energy, directly or through mechanical energy. In all of these designs and ideas the efficiency rises with increasing heat pressure. Equipment which operates on small temperature differences cannot be economical. The following plants of solar power stations have been suggested.

### Solar Power Farm

Dr. Meinel proposed building a large scale central station power plant using solar energy.<sup>48,49</sup> In his project, a Na-K solution would flow through miles of specially coated pipe and absorb solar energy. These coated pipes would be provided as a simple solar collector. Heating of Na-K solution would be accomplished by the pipe coating's high absorbance of solar radiation and low admittance of long-wave radiation. The hot Na-K solution would be used to generate steam for a Rankine cycle. Molten salt provides energy storage.



### Orbiting Solar Power Station

The idea of an orbiting solar power station has received much attention in the technical reports and the popular press.<sup>7,26,41-45,50</sup> The advantages of an orbiting solar cell power station are great--almost continual high energy solar radiation and no dust to cover the collector. The disadvantage is getting the station higher and hence the power lower. As envisioned, such a station would consist of a large solar cell array connected to a microwave converted by a 2 mile-long superconducting cable. The microwave energy is broadcast back to earth from a large antenna. On earth the microwave energy would be received by a large terrestrial-based antenna, reconverted to useful electrical energy, and put into an existing power grid.

The cost analysis is given by Glaser<sup>42</sup> and is based on a collector efficiency of 10%, a collector cost of \$1,000/kw, payload insertions of \$100/lb, \$10/kw for microwave generator, and \$40/kw for the cooling system. The projected costs for the system are as follows.

Insertion into synchronous orbit	\$ 45/kw
Earth receiving antenna	\$100/kw
Solar collector	\$300/kw
Microwave generator	\$ 50/kw
TOTAL	\$495/kw

For a  $10^7$  kw unit which is big enough to supply New York City, some typical sizes are:

Solar collector	$2.5 \times 10^6$ lb/ 25 square-mile
Antenna	$1 \times 10^6$ lb; 1 square-mile

Microwave equipment  $1 \times 10^6$  lb

Receiving antenna 36 square-mile

The block diagram of this system is shown in Figure 4.

#### High Altitude Solar Power Station

In this plan, a vast helium-filled mattress is floated at about 60,000 feet above the earth's surface.<sup>45</sup> This mattress, covered with silicon solar cells, would supply power to a large city's power grid. Transmission of this power would be by microwave transmission similar to the orbiting solar power station. The station would be anchored to the ground by steel cable. A one square mile array of 7% efficient solar cells, microwave equipment, and support equipment are estimated to weigh 10,000 tons and to produce 250,000 kw of electrical power. No cost estimates are given.

Other types of solar power stations such as solar wind equipment, solar thermoelectric equipment, solar pumps, solar engines with electric generators, etc. have been developed. However, due to small temperature differences, only low power can be obtained from those equipments.

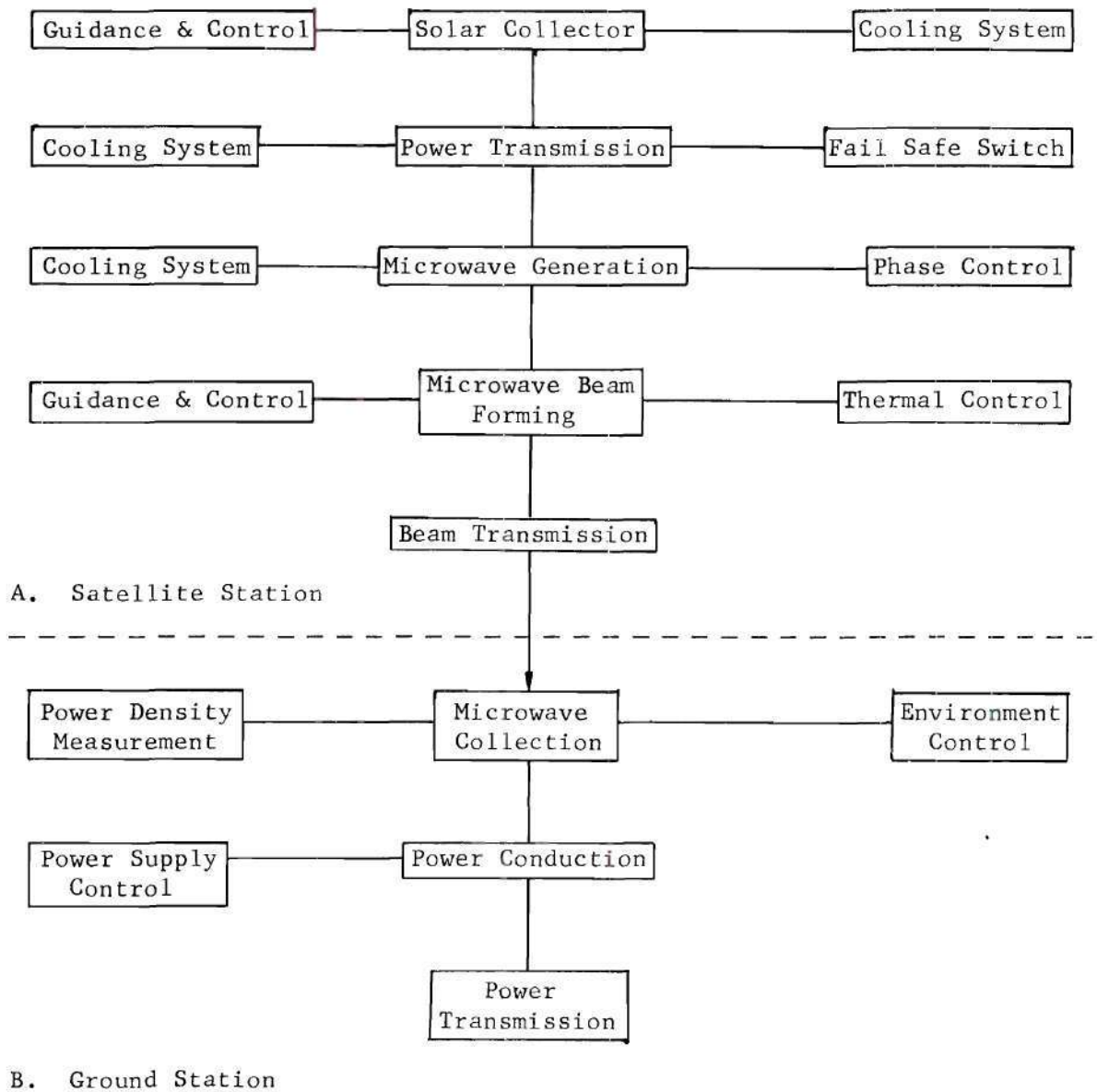


Figure 4. Major Components of an Orbiting Solar Power Station System<sup>50</sup>

## CHAPTER III

### ENERGY AVAILABILITY METHODS

#### Brief History of Energy Availability

Since the time of Carnot (1824), the concept of potential work-- in the sense of the maximum work which can be produced by a system or process--has been of concern to engineers dealing with power systems. This concept was inherent in the free energy and available energy functions of Von Helmholtz and Gibbs (1873). The concept was used by Darrieus (1930) who defined "thermodynamic efficiency" as being the quotient of the actual work obtained divided by the potential work for materials in steady flow.<sup>52</sup>

In 1941 Keenan formulated the following measure  $\Lambda$  of the potential work of closed system--a measure which he called "availability"<sup>53</sup>:

$$\Lambda = E + P_o V - T_o S - (E_o + P_o V_o - T_o S_o) \quad (3-1)$$

The subscript "o" denotes the closed system when it is in equilibrium with the surrounding medium so that the quantities  $P_o$ ,  $T_o$ , and  $(E_o + P_o V_o - T_o S_o)$  are constants. Keenan pointed to the property  $\Lambda$  as being "the maximum work which can be delivered to things other than the system and medium by the two unaided by any changes (except cyclic changes) in any external things."

For more general situations, Keenan (1951) wrote a balance equation for the term  $(E_o + P_o V - T_o S)$  which appears in equation (3-1). He pointed out that the use of the familiar Gibbs free energy function may be regarded as being a special case of this availability function. Further contributions to the availability concept were made by Rant (1956) and Gaggioli (1962). Rant introduced the term "exergy" which is widely used in Europe.<sup>54</sup>

In 1962, Evans presented the following measure  $\mathcal{A}$  of the potential work of a system--this measure at first having been called "available energy":

$$\mathcal{A} = E + P_o V - T_o S - \sum_c \mu_{co} N_c \quad (3-2)$$

In 1963, Evans used the symbol " $\mathcal{E}$ " for availability and called it "essergy." In view of these changes, the symbol " $\mathcal{E}$ " replaced the symbol " $\mathcal{A}$ " while the term "available energy" was replaced by the word "essergy."<sup>55</sup>

$$\mathcal{E} = E + P_o V - T_o S - \sum_c \mu_{co} N_c \quad (3-3)$$

It should be noted that equations (3-2) and (3-3) are identical.

#### Introductory Review of Energy Availability

Tribus and McIrvine has discussed the relationship between Equation (3-3) and thermodynamic information. For heat transfer  $Q$ , differentiation of equation (3-3) for a given environment yields  $d\mathcal{E} = dE - T_o dS$ , since  $dV$  and  $dN_c$  are zero when the only effect is heat transfer. Noting that



$dE = dQ$  while  $dS = dQ/T$  for reversible heat transfer at temperature  $T$ , we have

$$d\mathcal{E} = \frac{T - T_o}{T} dQ \quad (3-4)$$

The ratio  $(T - T_o)/T$  is denoted as the Carnot efficiency, which is conventionally looked upon as being the fraction of the heat  $dQ$  which is available as work. Thus the essergy formulation is seen to contain the Carnot principle within its framework.<sup>55</sup>

As a review of the Carnot principle, let us consider the following questions and answers. First, suppose an energy reservoir at  $1,000^\circ\text{R}$  gives up 1,000 Btu of heat. If the surrounding temperature is  $500^\circ\text{R}$ , how much of this heat can be converted into work by devices which operate cyclically? We answer this question by noting that the maximum work can be obtained by using a Carnot or Stirling reversible engine which receives 1,000 Btu at  $1,000^\circ\text{R}$  and rejects heat at  $500^\circ\text{R}$ . The efficiency of such an engine is 50%, so the answer to this question is that 500 Btu can be converted into work and this maximum amount of work can be obtained by using a Carnot engine, as shown in Figure 5.

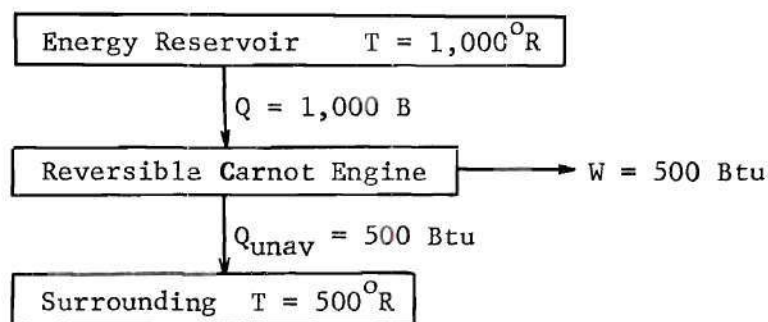


Figure 5. The Amount of Heat Converted into Work Using a Carnot Engine

Second, suppose that the 1,000 Btu is transferred from the reservoir at  $800^{\circ}\text{R}$  and then transferred to the surroundings at  $500^{\circ}\text{R}$ ; how much of this 1,000 Btu can be converted into work by cyclically operating devices? We use the Carnot reversible engine, but it's efficiency is

$$1 - \frac{T_{\text{low}}}{T_{\text{high}}} = 37.5\%; \text{ so the answer is 375 Btu converted into work.}$$

Those questions and their answers are important in engineering. Another question is related to the evaluation of the rejected energy by a system. One can use the available energy and unavailable energy concepts introduced by Rant<sup>\*</sup> to answer it.

Available energy is defined as that part of the heat added to a system which could be converted into work. Unavailable energy is defined as that part of the heat added to a system which could not be converted into work.

$$Q = Q_{\text{av}} + Q_{\text{unav}} \quad (3-5)$$

If heat  $Q$  is added to a system in a reversible process such as 1-2 in Figure 6, we can determine the available and unavailable energy parts of heat transferred.

An energy balance, with its attendant efficiency, is a basic tool for analyzing engineering systems. Such analyses, however, cannot evaluate the thermodynamic losses inherent in any real design and here availability concepts are more desirable.

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<sup>\*</sup>Rant<sup>54</sup> introduced the term "exergy" for available energy and "anergy"<sup>56</sup> for unavailable energy. This terminology is widely used in East Europe.

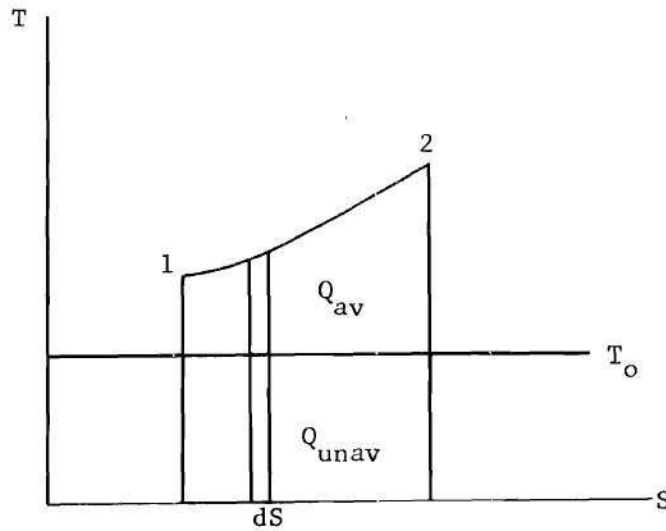


Figure 6. Available and Unavailable Energy Parts of Heat Transfer

$$Q_{\text{unav}} = \int_1^2 T_o dS = T_o (S_2 - S_1) \quad (3-6)$$

$$Q_{\text{av}} = \int_1^2 (T - T_o) dS = \int_1^2 T dS - Q_{\text{unav}} \quad (3-7)$$

Consider, first the energy balance dictated by the first law of thermodynamics for steady flow

$$Q + W = \Delta H \quad (3-8)$$

It means energy added to a system by heat  $Q$  and by work  $W$  are reflected exactly by the increase in enthalpy of the flow stream. Each of the various energies of equation (3-8) can be transformed in whole or in part into work by an ideal process (as restricted by the inevitable surrounding atmosphere at  $T_o, P_o$ ). This latent capacity to do work is called



the availability. Thus, an availability balance can be constructed to parallel equation (3-8).

Equation (3-9) states the availability transports added to a system by heat ( $\Delta B_q$ ) and by work ( $\Delta B_w$ ) are not reflected exactly by the availability of the flow system, since the second law of thermodynamics dictates that a loss of available energy,  $\theta$ , is inevitable for each real process.

$$\Delta B_q + \Delta B_w = \Delta B_f + \theta \quad (3-9)$$

$\Delta B_q$  : the maximum work capacity of a heat transfer  $\delta Q_i$  at temperature  $T_i$  equals work obtained from a reversible cycle operating between  $T_i$  and  $T_o$ .

$$\Delta B_q = \left(1 - \frac{T_o}{T_i}\right) \delta Q_i \quad \delta Q_i \text{ at } T_i \quad (3-10a)$$

$T_i$  is not necessarily constant.

$$\Delta B_q = \int_{T_1}^{T_2} \left(1 - \frac{T_o}{T_i}\right) dQ_i = Q_i - T_o \Delta S \quad (3-10b)$$

$\Delta B_w$  : the maximum work capacity of a work transport  $W$ .

$$\Delta B_w = W \quad (3-11)$$

$\Delta B_f$  : the availability for a reversible process of a steady state.

$$\Delta B_f = Q - T_o \Delta S + W \quad (3-12a)$$

$$\Delta B_f = H - T_o \Delta S \quad (3-12b)$$

$\theta$  : the irreversibility is calculated from equation (3-9).

Consider any various power systems which convert the available energy for fuel into work. All such systems are surrounded by the earth and, therefore,  $\Delta B_q = 0$  with entering and leaving streams at pressure  $P_o$  and temperature  $T_o$  of the surroundings. For a steady state reversible process under these restraints, roughly we get:

$$\Delta B_w = W_{\max} = H - T_o \Delta S \quad (3-13)$$

Equation (3-13) reveals that the maximum work from a power plant is either equal or greater or less than the heat of combustion,  $\Delta H$  depends on whether  $\Delta S$  is zero; negative or positive.

So we will discuss it in detail. The maximum work delivered by a steady flow system is the sum of that delivered by the system and that produced by a reversible heat engine, as shown in Figure 7.

$$\begin{aligned} \delta W_{\max} &= \delta W_{\text{system}} + \delta W_{\text{engine}} \\ &= \delta W_{\text{system}} + (T_o/T - 1) \delta Q \\ &= (\delta W - \delta Q) + T_o \frac{\delta Q}{T} \\ &= [(e_1 + p_1 v_1 - T_o s_1) - (e_2 + p_2 v_2 - T_o s_2)] \delta m + T_o \frac{\delta Q}{T} \end{aligned}$$

(continued)

$$\delta W_{\text{max useful}} = [(h_1 - T_o s_1) + \frac{1}{2g_c} (v_1^2 + gz_1) - (h_2 - T_o s_2) - \frac{1}{2g_c} (v_2^2 + gz_2)] \delta m$$

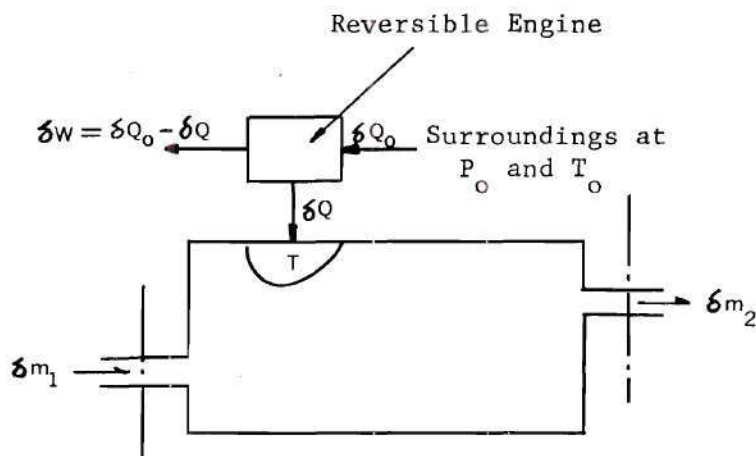


Figure 7. Composite System and Surroundings Producing Maximum Work

If heat is also exchanged with reservoir at temperature  $T_r$  and the changes of kinetic energy and potential energy can be neglected, the above equation may be modified by

$$\delta W_{\text{max}} = [(h_1 - T_o s_1) - (h_2 - T_o s_2)] \delta m + \delta Q_r \left( 1 - \frac{T_o}{T_r} \right) \quad (3-14)$$

The availability of a system in a given state is defined as the maximum useful work which can be obtained from the system and surroundings combination as the system goes from one state to the dead state\*

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\* Dead state: for a given initial state of a system which exchanges heat with the surroundings, the maximum possible amount of useful work can be done if the system goes to a state in which the pressure and the temperature equal those of the surroundings.

while exchanging heat only with the surroundings.

We know that  $b = h - T_o s$  is the availability. Substitute that into equation (3-14), then,

$$\delta W_{\max} = (b_1 - b_2) \delta m + Q_r \left( 1 - \frac{T_o}{T_r} \right) \quad (3-15)$$

By comparing the work done and availability change, a measure of the degree to which each processes approached to the ideal can be determined.

Now we will discuss the availability loss for a simplified power cycle as shown in Figure 8. In the steam power cycle, the increasing availability occurs in the boiler, superheater, and feed pump while decreases occur in the turbine, pipe, and heat exchanger.

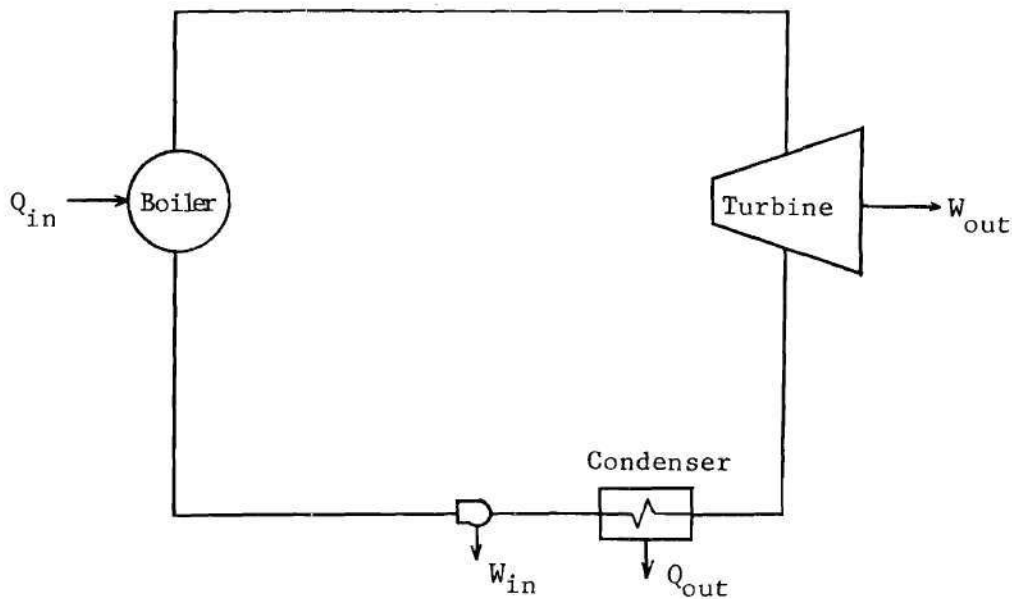


Figure 8. The Simplified Power Cycle

### Availability Loss of Heat Exchanger

Consider a boiler where the availability is transferred from the produce of combustion to the working fluid. The total availability supplied by the combustion product is

$$B_p = m_p \left[ (h_1 - T_o s_1) - (h_2 - T_o s_2) \right] \quad (3-16)$$

The availability gained by working fluid is

$$B_f = m_f \left[ (h_3 - T_o s_3) - (h_4 - T_o s_4) \right] \quad (3-17)$$

If we use the Carnot reversible engine between two temperatures, then

$$\Delta S_{\text{product}} + \Delta S_{\text{fluid}} = 0 \quad (3-18)$$

From equations (3-16), (3-17), and (3-18) we get

$$\begin{aligned} B_{\text{system}} &= \Delta B_p - \Delta B_f \\ \frac{B_{\text{system}}}{m_f} &= \frac{\Delta B_p}{m_f} - \frac{\Delta B_f}{m_f} \\ &= \frac{m_p}{m_f} \left[ (h_1 - h_2) - T_o (s_1 - s_2) \right] - \left[ (h_3 - h_4) - T_o (s_3 - s_4) \right] \\ &= \frac{m_p}{m_f} \Delta h_p - \Delta h_f \end{aligned} \quad (3-19)$$



For an ideal case, from the first law of thermodynamics we know that the amount of work of the system is exactly equal to the work produced by the reversible engine. Thus, this is no availability loss. For the actual case, however, the heat transfer process is irreversible, so irreversibility occurs and causes the availability loss.

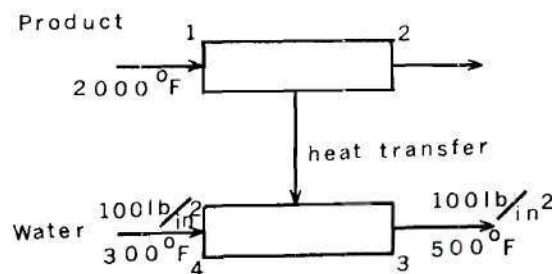
From the first law of thermodynamics,

$$m_p \times \Delta h_p = m_f \times \Delta h_f \quad (3-20)$$

$$\begin{aligned} \Delta B_{\text{system}} &= m_p \left[ (h_1 - h_2) - T_o (s_1 - s_2) \right] - m_f \left[ (h_3 - h_4) - T_o (s_3 - s_4) \right] \\ &= T_o (m_f \Delta s_f - m_p \Delta s_p) \end{aligned}$$

$$\frac{\Delta B_{\text{system}}}{m_f} = T_o \left( \Delta s_f - \frac{m_p}{m_f} \Delta s_p \right) \quad (3-21)$$

This case can be approached by considering a boiler. Heat is supplied by the products of combustion to steam, for example:



We can use the T-S diagram to explain the availability loss, as shown in Figure 9.

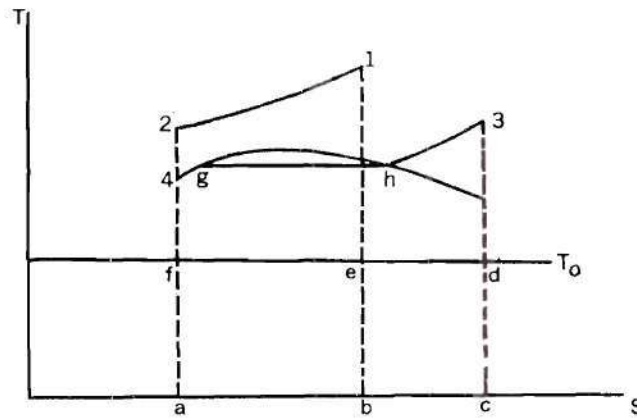


Figure 9. T-S Diagram for Explaining the Availability Loss

Line 1-2 represents the process of combustion product.

- I. Area 1-2-f-e-1 represents the reversible work for the given change in state of these products.
- II. Area 1-2-a-b-1 represents the heat transfer from the combustion products.
- III. Area 4-g-h-3-c-a-4 represents the heat transferred to the water.  
This area must be equal to Area I.
- IV. Area 4-g-h-3-d-f-4 represents the reversible work for the given change in state of the water.

$$\begin{aligned}
 W_{\text{net}} &= \text{Area II} - \text{Area IV} \\
 &= \text{Area of } b-c-d-e \\
 &= T_o (S)_{\text{net}}
 \end{aligned}$$

Since the actual work is zero, this area also represents the irreversibility--the availability loss.

The same analysis can be applied to the condenser and feed-water heater.

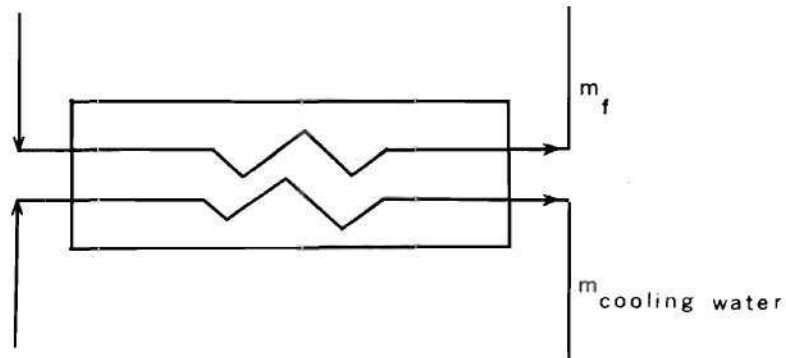


Figure 10. The Heat Exchanger

$$\frac{\Delta B_{\text{loss}}}{m_f} = T_o \left( \Delta s_f - \frac{m_{\text{cooling water}}}{m_f} \Delta s_{\text{cooling water}} \right) \quad (3-22)$$

#### Availability Loss Due to Turbine, Pump, and Pipe

The energy loss in a turbine is primarily due to the working fluid passing through the turbine and pump. Friction effect and heat transfer to the surroundings are important causes of availability loss. For an open system, the entropy change can be expressed as

$$dS_{\text{e}} \geq \frac{\delta Q}{T} + \delta m_i s_i - \delta m_o s_o \quad (3-23)$$

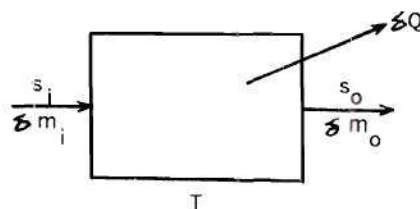


Figure 11. An Open System

If we introduce the concept of lost work ( $W_L$ )<sup>57</sup> into equation (3-23) then

$$TdS = dU + dW + dW_L \quad (3-24)$$

$$\delta Q = dU + \delta W$$

$$dS = \frac{\delta Q + \delta W_L}{T} + \delta m_i s_i + \delta m_o s_o$$

For a steady state system  $\delta m_i = \delta m_o = \delta m$ , and  $dS = 0$ , then

$$\delta W_L = T(s_e - s_i) - \delta Q \quad (3-25)$$

Since the lost work is in the form of mechanical work and is not a form of heat before the process, the decrease in lost work is equal to the availability loss. This can be proved as follows:

From the first law of thermodynamics

$$dh + \frac{dV^2}{2g_c} + \frac{gdZ}{g_c} + \delta W - \delta Q = 0 \quad (3-26)$$

Since

$$Tds = dh - vdp$$

$$dh = Tds + vdp \quad (3-27)$$

$$= du + dw + dw_L + vdp$$

$$= \delta q + \delta w_L + vdp$$

Substituting equation (3-27) into equation (3-26), and we know that there is no work done,  $\delta W = 0$ .

$$\delta w_L = -v dp - \frac{dV^2}{2g_c} - \frac{g dZ}{g_c} = \delta b_{\text{loss}} \quad (3-28)$$

When the work is lost, heat is produced and transferred to the system, then the heat gain is

$$\begin{aligned} \delta Q_{\text{gain}} &= \delta W_L - (-\delta Q) \\ &= T \delta m (s_o - s_i) \end{aligned} \quad (3-29)$$

However, only part of the heat gained is available energy. This amount is equal to

$$\begin{aligned} \Delta B_{\text{gain}} &= \left(1 - \frac{T_o}{T}\right) \delta Q_{\text{gain}} \\ &= (T - T_o) m (s_o - s_i) \end{aligned} \quad (3-30)$$

From equations (3-25) and (3-27) we get

$$\begin{aligned} \Delta B_{\text{net}} &= \Delta B_{\text{loss}} + \Delta B_{\text{gain}} \\ &= T \delta m (s_o - s_i) - \delta Q + (T - T_o) \delta m (s_o - s_i) \\ &= T_o \delta m (s_o - s_i) - \delta Q \end{aligned} \quad (3-31)$$

Equation (3-26) is the net availability loss for an open system.

$$\Delta B_{\text{net loss}} = T_o \delta m (s_o - s_i) - \delta Q \quad (3-32)$$



Equation (3-27) is the availability loss due to turbine, pump, and pipe.

#### The Principle of Constant Effectiveness

Kreith<sup>58</sup> pointed out that the effectiveness ( $e = \frac{2_{\text{actual}}}{2_{\text{Carnot}}}$ ) tends to remain constant for a given cycle over a wide range of operating conditions. Here we show demonstrated the principle of constant effectiveness via Rankine cycle.

Rankine cycle consists of four main process, as shown in Figure 12. The available energy increases at pump and boiler while decreasing at turbine and condenser. The following derivation is based on this cycle, but the results may be extended to more general systems (including re-generation and reheat) as shown by Chou.<sup>59</sup>

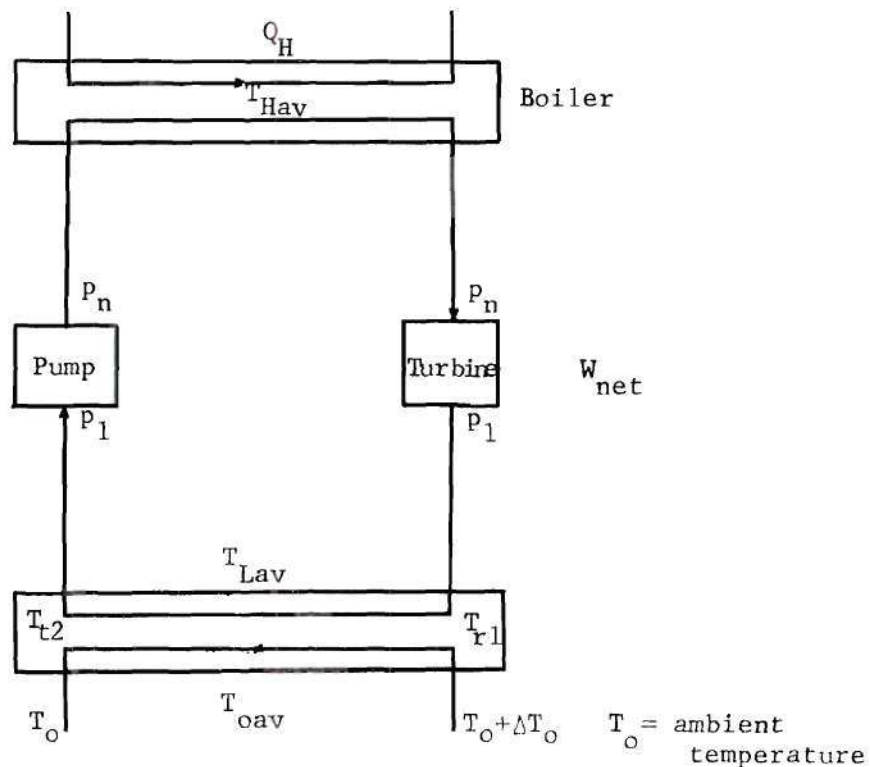


Figure 12. Rankine Cycle

The rate of available energy gained by water in the boiler is

$$\begin{aligned} B_{in} &= \dot{m} \left[ (h_o - T_o s_o) - (h_i - T_i s_i) \right] \\ &= \dot{m} \left[ (h_o - h_i) - T_o (s_o - s_i) \right] \end{aligned} \quad (3-33)$$

Since  $Q_H = \dot{m} (h_o - h_i)$

And  $\dot{m} (s_o - s_i) = \int \frac{dQ_H}{T_H} \approx \frac{Q_H}{T_{Hav}}$

Thus  $B_{in} = Q_H \left( 1 - \frac{T_o}{T_{Hav}} \right)$  (3-33a)

Now we define the effectiveness as  $e = \frac{W_{net}}{B_{in}}$ , since the part of the heat supplied to steam is available energy.

$$e = \frac{W_{net}}{Q_H \left( 1 - \frac{T_o}{T_{Hav}} \right)} \quad (3-34)$$

The effectiveness of a power plant is always bigger than its efficiency.

$$e_{\text{power plant}} > \eta_{\text{power plant}}$$

The effectiveness may be equal to efficiency only if  $T_{Hav}$  is infinite or if  $T_o$  is equal to absolute zero. It is practically impossible to get such temperatures.

The availability supplied to the working fluid is steam, by the solar radiation is equal to

$$\begin{aligned}
 \dot{B}_{in} &= Q_H - T_o \int \frac{\delta Q_H}{T_H} \\
 &= Q_H - T_o \frac{Q_H}{T_{Hav}} \\
 &= Q_H \left( 1 - \frac{T_o}{T_{Hav}} \right)
 \end{aligned}$$

Where

$$T_{Hav} \simeq \frac{Q_H}{\int \frac{\delta Q_H}{T_H}}$$

The output of a turbine is equal to

$$\begin{aligned}
 \dot{W}_t &= \dot{m} \left( \int_{p_1}^{p_h} v_t (dp)_s - \Delta h_{t,loss} \right) \\
 &= \eta_t \dot{m} \int_{p_1}^{p_h} v_t (dp)_s
 \end{aligned} \tag{3-35}$$

Where  $\eta_t$  is turbine efficiency, and  $\dot{m} \int_{p_1}^{p_h} v (dp)_s$  is the reversible adiabatic work of the turbine.

The pump's work is

$$\begin{aligned}
 \dot{W}_{pump} &= \frac{\dot{m}}{\eta_{pump}} \int_{p_1}^{p_h} v_{pump} (dp)_s \\
 &= \dot{m} \left( \int_{p_1}^{p_h} v_{pump} (dp)_s - \Delta h_{pump,loss} \right)
 \end{aligned} \tag{3-36}$$

Therefore

$$\dot{W}_{net} = \dot{W}_t - \dot{W}_{pump} \tag{3-37}$$

Since the availability is  $B_{in}$ .<sup>55</sup>

$$\dot{B}_{in} = \dot{W}_{net} + T_o(\Delta S_t + \Delta S_{pump} + \Delta S_{con} + \Delta S_{pipe}) + \dot{B}_{out} \quad (3-39)$$

$$e = \frac{2\dot{m} \int_{p_1}^{p_h} v_t(dp)_s - \frac{\dot{m}}{2\dot{m}_{pump}} \int_{p_1}^{p_h} v_p(dp)_s}{\dot{m} \left( \int_{p_1}^{p_h} v_t(dp)_s - \Delta h_{t,loss} \right) + T_o(\Delta S_t + \Delta S_{pump} + \Delta S_{con} + \Delta S_{pipe}) + \dot{B}_{out} - \dot{W}_{pump}} \quad (3-40)$$

The  $\Delta h_{t,loss}$  in equation (3-35) can be found to be related to the entropy increase in a turbine. Since most turbines try to expand the steam to a low pressure, the exhaust steam condition is normally in the two phase region or near the saturation line. Figure 13 shows this on a Mollier diagram.

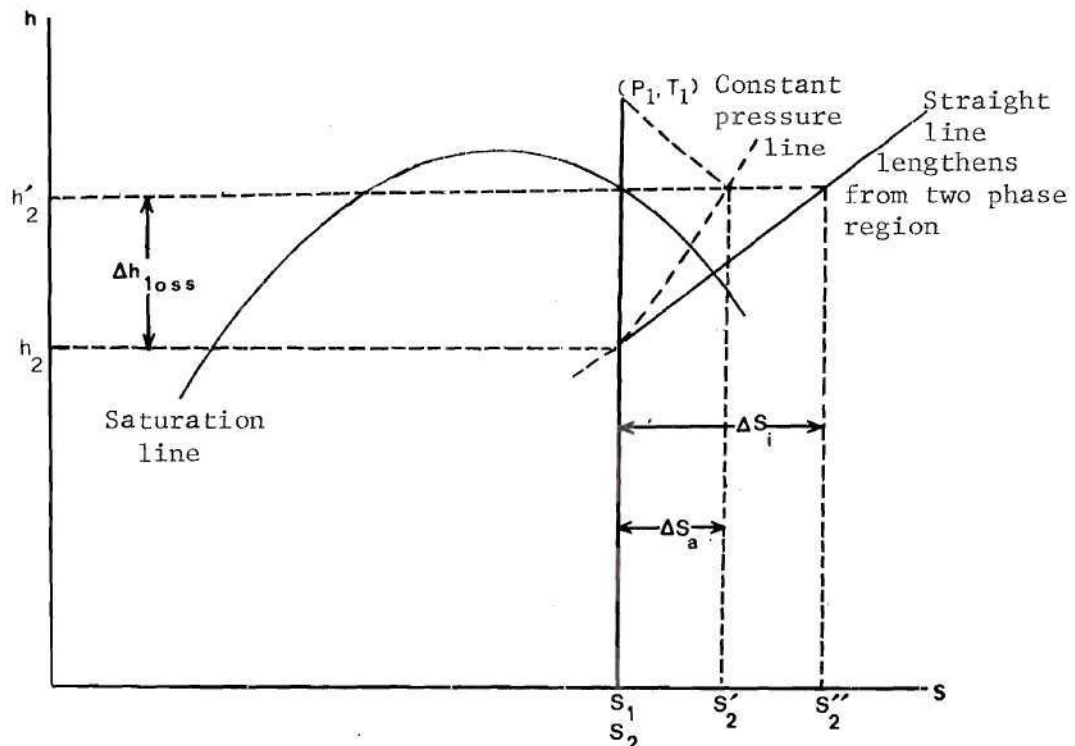


Figure 13. Mollier Diagram

$$\Delta h_{\text{loss}} = h_2' - h_2$$

$$dh = Tds + vdp \quad (3-41)$$

$$= \left( \frac{\partial h}{\partial s} \right)_p ds + \left( \frac{\partial h}{\partial p} \right)_v dp$$

$$T = \left( \frac{\partial h}{\partial s} \right)_p$$

It is known that the saturation temperature is fixed in the two phase region by a constant pressure. Therefore, in the two phase region we get

$$\Delta s_{t,i} = \frac{\Delta h_{t,\text{loss}}}{T_2} \quad (3-42a)$$

or 
$$\Delta h_{t,\text{loss}} = \Delta s_{t,i} \times T_2 \quad (3-42b)$$

The above equations are exactly valid for the exhaust steam in the two phase region. Furthermore, they still hold approximately in the superheated region near the saturation line. Although in the real case the constant pressure line in the superheated region is a slight upward curve, we can check it with an example.

Assume the inlet pressure and temperature of a turbine are

$$P_h = 3600 \text{ psi}$$

$$T_h = 1500^\circ\text{F}$$

$$P_1 = 1 \text{ psi}$$

The turbine efficiency is 0.7



Therefore State 1:

$$P_1 = 3600 \text{ psi}$$

$$T_1 = 1500^\circ\text{F}$$

$$h_1 = 1755.9 \text{ Btu/lbm}$$

$$s_1 = 1.6654 \text{ Btu/lbm-}^\circ\text{R}$$

State 2:

$$P_2 = 1 \text{ psi}$$

$$T_2 = 101.7^\circ\text{F}$$

Since  $s_2 = s_1 = 1.6654 \text{ Btu/lbm-}^\circ\text{R}$

And  $s_2 = s_{2f} + x \cdot s_{fg}$

$$1.6654 = 0.13266 + x \cdot (1.8453)$$

$$x = 0.8284$$

$$\begin{aligned} h_2 &= h_{2f} + x \cdot h_{2fg} \\ &= 69.74 + 0.8384 \times 1036 \\ &= 927.96 \text{ Btu/lbm} \end{aligned}$$

$$\begin{aligned} \Delta h &= 1755.9 - 927.96 \\ &= 827.94 \text{ Btu/lbm} \end{aligned}$$

$$\begin{aligned} \eta_t \Delta h &= 0.7 \times 827.94 \\ &= 579.29 \text{ Btu/lbm} \end{aligned}$$

$$\begin{aligned} h_2' &= h_1 - \eta_t \Delta h \\ &= 1755.9 - 579.29 \\ &= 1176.61 \text{ Btu/lbm} \end{aligned}$$

From the steam table we find

$$T_2' = 258.3^\circ\text{F}$$

$$s_2' = 2.0894 \text{ Btu/lbm-}^\circ\text{R}$$

Now we calculate by using equation (3-42a) to find entropy change.

$$\begin{aligned}\Delta s_{t,i} &= \frac{\Delta h_{t,loss}}{T_2} \\ &= \frac{827.94 - 579.29}{101.7 + 460} \\ &= 0.4427 \text{ Btu/lbm-}^{\circ}\text{R}\end{aligned}$$

$$\begin{aligned}\Delta s_{t,a} &= s_2' - s_1 \\ &= 2.0894 - 1.6654 \\ &= 0.425 \text{ Btu/lbm-}^{\circ}\text{R}\end{aligned}$$

Therefore the error percentage is

$$\% = \frac{0.4427 - 0.425}{0.425} = 4.16\%$$

If we check the point at  $h_2' = 1176.16 \text{ Btu/lbm}$ ,  $s_2' = 2.0894 \text{ Btu/lbm-}^{\circ}\text{R}$ . On a Mollier chart, we find that this point is a slight distance away from the saturation line. However, the error between these two methods is limited to 4.16%. Therefore, we can roughly conclude

"  $\Delta s_{t,i} = \frac{\Delta h_{t,loss}}{T_2}$  " holds pretty well for a turbine whose exhaust steam is the superheated region near the saturation line.

When  $\Delta s_i$  is calculated using equation (3-42a) it is always larger than  $\Delta s_a$  for real cases (see Figure 12)

Now we substitute equation (3-42a) into equation (3-40).

Then

$$\begin{aligned}
e &= \frac{\eta_t \dot{m} \int_{p_1}^{p_h} v_t(dp)_s - \frac{\dot{m}}{\eta_{\text{pump}}} \int_{p_1}^{p_h} v_{\text{pump}}(dp)_s}{\left[ \dot{m} \int_{p_1}^{p_h} v_t(dp)_s - T_{L1} \Delta S_t \right] + T_o (\Delta S_t + \Delta S_{\text{pump}} + \Delta S_{\text{con}} + \Delta S_{\text{pipe}}) + \dot{B}_{\text{out}} - \dot{W}_{\text{pump}}} \\
&= \frac{\eta_t \dot{m} \int_{p_1}^{p_h} v_t(dp)_s - \frac{\dot{m}}{\eta_{\text{pump}}} \int_{p_1}^{p_h} v_{\text{pump}}(dp)_s}{\dot{m} \int_{p_1}^{p_h} v_t(dp)_s + (T_o - T_{L1}) \Delta S_t + T_o (\Delta S_{\text{pump}} + \Delta S_{\text{con}} + \Delta S_{\text{pipe}}) + \dot{B}_{\text{out}} - \dot{W}_{\text{pump}}} \\
&= \frac{\eta_t \dot{m} \int_{p_1}^{p_h} v_t(dp)_s - \frac{\dot{m}}{\eta_{\text{pump}}} \int_{p_1}^{p_h} v_{\text{pump}}(dp)_s}{\dot{m} \int_{p_1}^{p_h} v_t(dp)_s + \frac{T_o - T_{L1}}{T_{L1}} (1 - \eta_t) \dot{m} \int_{p_1}^{p_h} v_t(dp)_s + T_o (\Delta S_{\text{pump}} + \Delta S_{\text{con}} + \Delta S_{\text{pipe}}) + \dot{B}_{\text{out}} - \dot{W}_{\text{pump}}}
\end{aligned}$$

Dividing the numerator and denominator by  $\dot{m} \int_{p_1}^{p_h} v_t(dp)_s$ , one gets

$$\begin{aligned}
e &= \frac{\eta_t - \frac{\dot{m} \int_{p_1}^{p_h} v_{\text{pump}}(dp)_s}{\eta_{\text{pump}} \dot{m} \int_{p_1}^{p_h} v_{\text{pump}}(dp)_s}}{1 - \frac{T_o - T_{L1}}{T_{L1}} (1 - \eta_t) + \frac{T_o (\Delta S_{\text{pump}} + \Delta S_{\text{con}} + \Delta S_{\text{pipe}}) + \dot{B}_{\text{out}} - \dot{W}_{\text{pump}}}{\dot{m} \int_{p_1}^{p_h} v_t(dp)_s}} \quad (3-43)
\end{aligned}$$

For simplification, we can neglect pump work and the entropy change in pump and pipe, because the pump work is very small in comparison to the total output of a power plant. Since the pump work is negligible, the entropy change in the pump is also negligible. The same is true for the pipe. Therefore,

$$\frac{\int_{p_1}^{p_h} v_{\text{pump}}(dp)_s}{\eta_{\text{pump}} \int_{p_1}^{p_h} v_{\text{pump}}(dp)_s} \cong 0 \quad \Delta S_{\text{pump}} \cong 0 \quad \Delta S_{\text{pipe}} \cong 0$$

So equation (3-43) becomes

$$e = \frac{\eta_t}{1 + \left( \frac{T_o - T_{L1}}{T_{L1}} \right) (1 - \eta_t) + \frac{T_o \Delta S_{con} + \dot{B}_{out}}{\int_{p_1}^{p_h} v_t (dp)_s}} \quad (3-44)$$

because the entropy change for a heat exchanger is

$$\begin{aligned} S &= \frac{Q}{T_b} - \frac{Q}{T_a} \\ &= Q \left( \frac{T_a - T_b}{T_a T_b} \right) \end{aligned}$$

$T_a$  and  $T_b$  are the average temperature of cooling water and heated water.

Figure 14 is the schematic diagram of a heat exchanger.

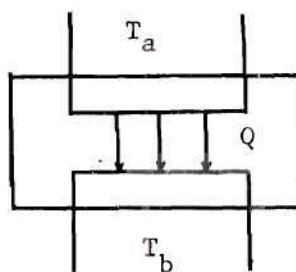


Figure 14. Schematic Diagram of a Heat Exchanger

The entropy increase in a condenser is

$$\Delta S_{con} = \dot{Q}_L \left( \frac{\Delta T_L}{T_{Lav} \cdot T_{oav}} \right) \quad (3-45)$$

The available energy leaving the condenser is

$$\begin{aligned} \dot{B}_{out} &= \dot{Q}_L - T_o \int_{T_o}^{T_o + T_{cooling}} \frac{\delta \dot{Q}_L}{T} \\ &= \dot{Q}_L \left(1 - \frac{T_o}{T_{oav}}\right) \end{aligned} \quad (3-46)$$

Substituting equation (3-45) into equation (3-46) gives

$$e = \frac{\eta_t}{1 + \left(\frac{T_o - T_{L1}}{T_{L1}}\right)(1 - \eta_t) + \frac{\dot{Q}_L \left[ \frac{\Delta T_L \cdot T_o}{T_{Lav} T_{oav}} + \frac{T_{oav} - T_o}{T_{oav}} \right]}{\dot{m} \int_{p_1}^{p_h} v_t (dp)_s} \quad (3-47)$$

From the first law of thermodynamics we know

$$\dot{Q}_L = \dot{Q}_H - \eta_t \dot{m} \int_{p_1}^{p_h} v_t (dp)_s + \dot{W}_{pump} \quad (3-48)$$

Comparing  $\dot{W}_{pump}$  and  $\dot{W}_{turbine}$ , we can neglect the  $\dot{W}_{pump}$ .

$$\dot{Q}_L = \dot{Q}_H - \eta_t \dot{m} \int_{p_1}^{p_h} v_t (dp)_s \quad (3-49)$$

$$\frac{\dot{Q}_L}{\dot{m} \int_{p_1}^{p_h} v_t (dp)_s} = \frac{\dot{Q}_H}{\dot{m} \int_{p_1}^{p_h} v_t (dp)_s} - \eta_t \quad (3-50)$$

Since

$$\frac{\dot{Q}_H}{\dot{m} \int_{p_1}^{p_h} v_t (dp)_s} = \frac{1}{\frac{W_{t, ideal}}{\dot{Q}_H}}$$

$$= \frac{1}{1 - \frac{T_{Lav}}{T_{Hav}}}$$

(continued)



$$\frac{\dot{Q}_H}{\dot{m} \int_{p_1}^{p_h} v_t (dp)_s} \approx \frac{T_{Hav}}{T_{Hav} - T_{Lav}}$$

Then equation (3-50) becomes

$$\begin{aligned} \frac{\dot{Q}_H}{\dot{m} \int_{p_1}^{p_h} v_t (dp)_s} &= \frac{T_{Hav}}{T_{Hav} - T_{Lav}} - \eta_t \\ e &= \frac{\eta_t}{1 + \left( \frac{T_o - T_{L1}}{T_{L1}} \right) (1 - \eta_t) \left[ \frac{T_{Hav}}{T_{Hav} - T_{Lav}} - \eta_t \right] \left[ \frac{\Delta T_L \cdot T_o}{T_{Lav} T_{oav}} + \frac{T_{oav} - T_o}{T_{oav}} \right]} \end{aligned} \quad (3-51)$$

Let  $T_{oav} = T_o + \Delta T_o / 2$ ; then

$$\begin{aligned} e &= \frac{\eta_t}{1 + \left( \frac{T_o - T_{L1}}{T_{L1}} \right) (1 - \eta_t) \left[ \frac{T_{Hav}}{T_{Hav} - T_{Lav}} - \eta_t \right] \left[ \frac{\Delta T_L \cdot T_o}{T_{Lav} T_{oav}} + \frac{\Delta T_o}{2 T_{oav}} \right]} \\ e &= \frac{\eta_t}{1 + \left( \frac{T_o - T_{L1}}{T_{L1}} \right) (1 - \eta_t) \left[ \frac{T_{Hav}}{T_{Hav} - T_{Lav}} - \eta_t \right] \left[ \frac{\Delta T_L \cdot T_o}{T_{Lav} T_{oav}} + \frac{\Delta T_o}{2 T_o + \Delta T_o} \right]} \end{aligned} \quad (3-52)$$

For convenience we can write in this way

$$e = \frac{t}{1 + c} \quad (3-53)$$

Where  $c$  is the correction factor.

$$c = \frac{T_o - T_{L1}}{T_{L1}} (1 - \eta_t) + \left[ \frac{T_{Hav}}{T_{Hav} - T_{Lav}} - \eta_t \right] \left[ \frac{\Delta T_L \cdot T_o}{T_{Lav} T_{oav}} + \frac{\Delta T_o}{2 T_o + \Delta T_o} \right]$$

$T_L$  and  $T_{Lav}$  are almost equal because most heat rejected in the condenser is latent heat of steam, i.e., a phase change takes place in the condenser so that the temperature is constant for the given pressure, i.e.,  $T_L \approx T_{Lav}$ .

In most cases the condenser has a typical terminal temperature difference of 5 to 10°F. If we take the average value of the terminal temperature difference to be 7.5°F, then

$$T_L = \frac{27.5 - 7.5}{\ln \frac{27.5}{7.5}} = 15^\circ\text{F}$$

Now we will give some data using equation (3-53), checking to see whether or not the effectiveness is constant during the Rankine cycle. During this process, substituting these values into equations (3-52) and (3-53), we get the results tabulated in Tables 3, 4, and 5. Also we plot  $e$  vs.  $T_{Hav}$  in Figures 15, 16, and 17. From these results we find effectiveness decreased very little as  $T_{Hav}$  was decreased 50°F and, therefore, we can say the effectiveness is virtually constant. We may call this principle the principle of constant effectiveness. Although this result was derived using the simple Rankine cycle, Chou<sup>59</sup> has shown that the result remains basically the same when the effects of regeneration, reheat, and topping are included.

Table 3. The Result of Equation (3-53) for Given Data  
with Constants  $\Delta T_L$ ,  $\Delta T_o$ , and  $T_{oav}$

$\eta_t$	$T_L$ ( $^{\circ}R$ )	$T_o$ ( $^{\circ}R$ )	$\Delta T_L$	$\Delta T_o$	$T_{Hav}$ ( $^{\circ}R$ )	$T_{oav}$	c	e
0.75	550	535	15	20	1500	545	0.0306	0.7277
0.75	550	535	15	20	1450	545	0.0320	0.7267
0.75	550	535	15	20	1400	545	0.0337	0.7255
0.75	550	535	15	20	1350	545	0.0355	0.7243
0.75	550	535	15	20	1300	545	0.0374	0.7229
0.75	550	535	15	20	1250	545	0.0399	0.7212
0.75	550	535	15	20	1200	545	0.0422	0.7193
0.75	550	535	15	20	1150	545	0.0458	0.7172
0.75	550	535	15	20	1100	545	0.0496	0.7146
0.75	550	535	15	20	1000	545	0.0596	0.7078

Table 4. The Result of Equation (3-53) for Some Given Data

$\eta_t$	$T_L$	$T_o$	$\Delta T_L$	$\Delta T_o$	$T_{oav}$	$T_{Hav}$	c	$\eta$
0.75	540	525	10	15	532.5	1500	0.0193	0.7358
0.75	540	525	11	16	533	1350	0.0252	0.7316
0.75	540	525	12	17	533.5	1200	0.0354	0.7244
0.75	540	525	13	18	534	1100	0.04205	0.7197
0.75	540	525	14	19	534.5	1065	0.0483	0.7154

Table 5. The Result of Equation (3-53) for Some Given Data  
Such as Various  $\eta_t$  and  $T_{Hav}$

$\eta_t$	$T_L$	$T_o$	$\Delta T_L$	$\Delta T_o$	$T_{oav}$	$T_{Hav}$	c	e
0.85	540	525	15	20	535	1500	0.0214	0.8322
0.83	540	525	15	20	535	1450	0.0221	0.8121
0.80	540	525	15	20	535	1300	0.0272	0.7788
0.77	540	525	15	20	535	1200	0.0307	0.7470
0.75	540	525	15	20	535	1100	0.0439	0.7185
0.73	540	525	15	20	535	1050	0.0403	0.6921
0.71	540	525	15	20	535	1000	0.0446	0.6797

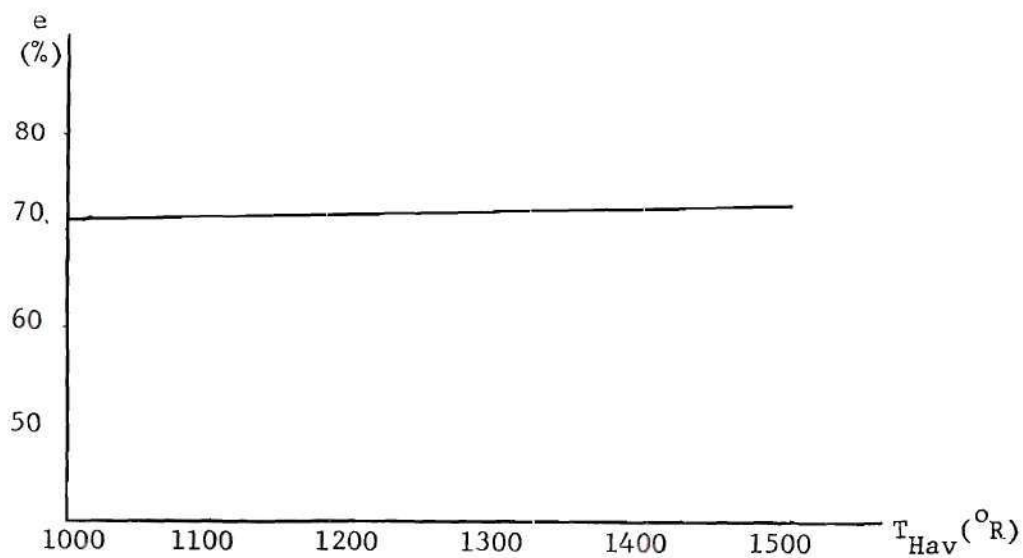


Figure 15. The Result of Equation (3-53) with Constants  
 $\Delta T_L$ ,  $\Delta T_o$ , and  $T_{oav}$  (e vs.  $T_{hav}$ )

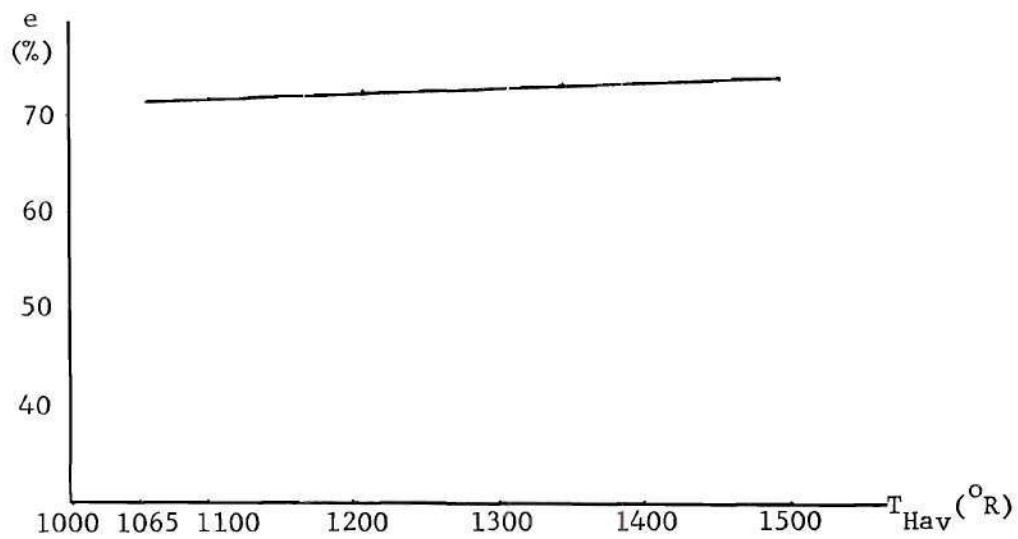


Figure 16. The Result of Equation (3-53) with Variable Data (e vs.  $T_{Hav}$ )

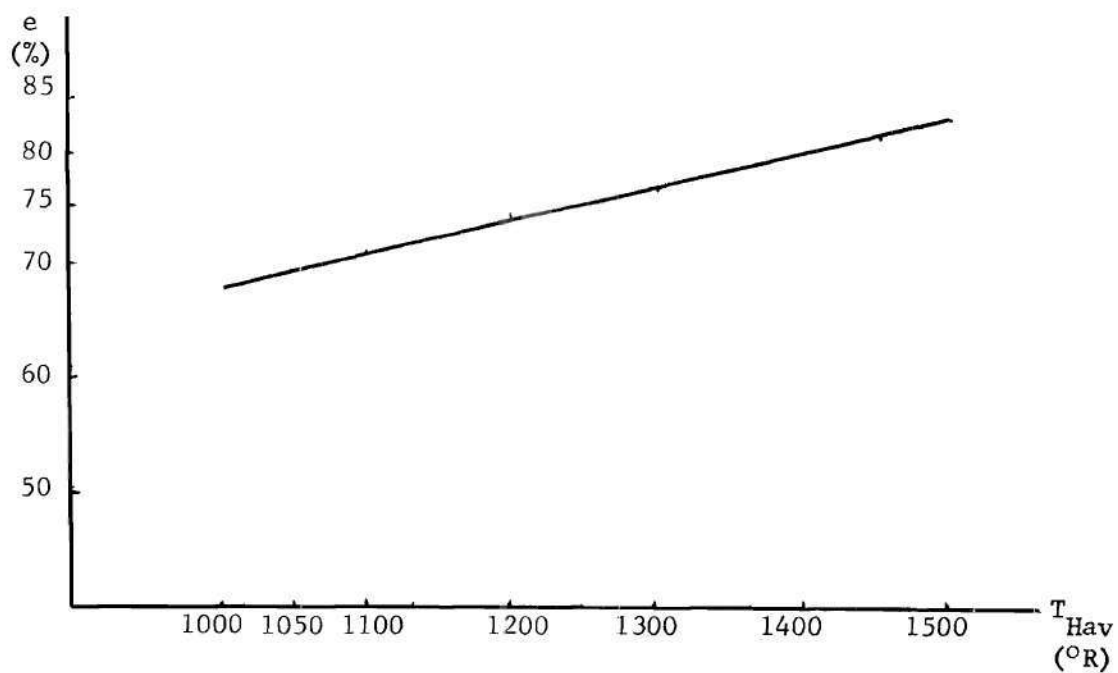


Figure 17. The Effectiveness of Variable  $\eta_t$  (e vs.  $T_{Hav}$ )



### Optimum Solar Boiler Temperature

From the previous discussion we have derived the result that effectiveness of a power plant is virtually constant with respect to  $T_{Hav}$  (average temperature of boiler). Now we will use this result to find the optimum temperature of a solar boiler for the following conditions.

- 1)  $C_{power}$  = cost per unit of work (cent/kw-hr)
- 2)  $\dot{C}_{plant}$  = cost of plant, taken to be constant for this simple analysis
- 3)  $W$  = output; is constant, i.e. fixed demand.

Because

$$\begin{aligned}
 C_{power} &= \frac{\dot{C}_{plant}}{\dot{W}} + \frac{\dot{C}_{collector}}{\dot{W}} \\
 &= \frac{\dot{C}_{plant}}{\dot{W}} + \frac{\dot{C}_{collector}}{\dot{W}_{max} \cdot e}
 \end{aligned} \tag{3-54}$$

Where

$$e = \frac{\dot{W}}{\dot{W}_{max}}$$

Taking  $e$  to be constant via the principle of constant effectiveness found in the preceding section, minimum  $C_{power}$  then corresponds to minimum

$$\frac{\dot{C}_{collector}}{\dot{W}_{max}}$$

Where  $\frac{\dot{C}_{collector}}{\dot{W}_{max}}$  = cost per unit of available energy

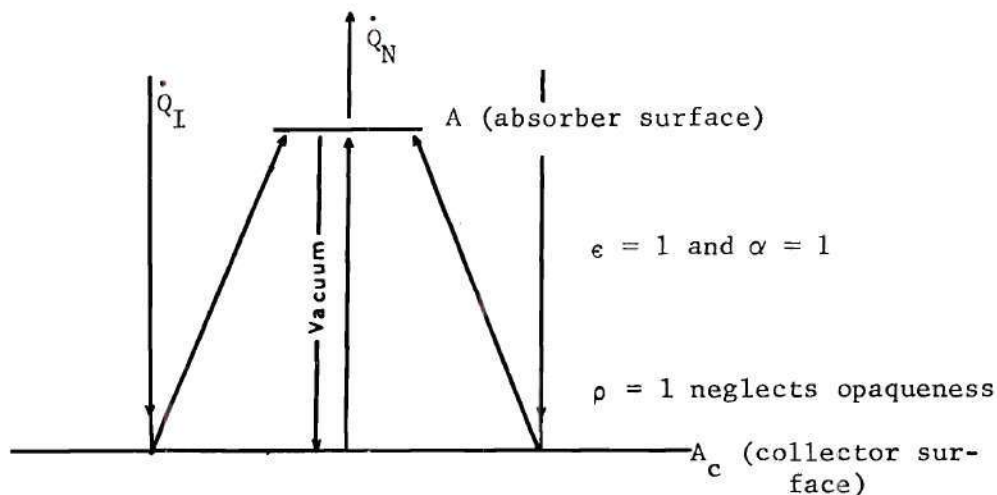


Figure 18. Outline of Collector and Absorber

In Figure 18 we assume, for simplicity, that the space between collector and absorber is a vacuum; transmissivity of collector equal to one; the emissivity and absorptivity of the absorber each equal to one and neglecting opaqueness. Then

$$\dot{q}_I = \frac{\dot{Q}_I}{A_c} \quad (3-55)$$

$$X = \frac{A_c}{A} \text{ (concentration ratio)} \quad (3-56)$$

$$\dot{Q}_N = \dot{q}_{\text{net}} A$$

$$\frac{\dot{Q}_N}{A} = \dot{q}_n \quad (3-57)$$

With a conservative assumption that all of the re-radiated energy is lost,

thus neglecting return radiation and shape factors, the energy balance can be written as follows

$$\dot{Q}_I = \dot{Q}_N + A\sigma T_H^4 \quad (3-58)$$

$$A_c \dot{q}_I = A \dot{q}_N + A\sigma T_H^4$$

$$X \dot{q}_I = \dot{q}_n + \sigma T_H^4 \quad (3-59)$$

Define  $\eta_{\text{collector}} = \frac{\dot{q}_n}{X \dot{q}_I} = \frac{\dot{Q}_N}{\dot{Q}_I} \quad (3-60)$

So 
$$= \frac{X \dot{q}_I - \sigma T_H^4}{X \dot{q}_I}$$

$$= 1 - \frac{\sigma T_H^4}{X \dot{q}_I} \quad (3-61)$$

The maximum temperature  $T_H$  occurs when  $\dot{q}_n = 0$  or when  $\eta_{\text{collector}} = 0$ .

Thus

$$X \dot{q}_I = \sigma T_{\text{max}}^4 \quad (3-62)$$

Substituting equation (3-62) into equation (3-61) yields

$$\eta_{\text{collector}} = 1 - \left( \frac{T_H}{T_{\text{max}}} \right)^4 \quad (3-63)$$

From the second law of thermodynamics,

$$T_{\max} = T_{\text{sun's surface}} \approx 10,500^{\circ}\text{R}$$

Now consider minimizing  $\dot{C}_{\text{collector}}/\dot{W}_{\max}$

$$\begin{aligned} \frac{\dot{C}_{\text{collector}}}{\dot{W}_{\max}} &= \frac{\dot{C}_{\text{collector}}}{\dot{Q}_N \frac{T_H - T_o}{T_H}} & (3-64) \\ &= \frac{\dot{C}_{\text{collector}}}{\dot{Q}_I \frac{\dot{Q}_N}{\dot{Q}_I} \frac{T_H - T_o}{T_H}} \\ &= \frac{\dot{C}_{\text{collector}}}{\dot{Q}_I \eta_{\text{collector}} \frac{T_H - T_o}{T_H}} \\ &= \frac{\dot{C}_{\text{collector}}}{A_c \dot{q}_I \eta_{\text{collector}} \frac{T_H - T_o}{T_H}} \\ &= \frac{1}{\left[1 - \left(\frac{T_H}{T_{\max}}\right)^4\right] \frac{T_H - T_o}{T_H}} \frac{\dot{C}_{\text{collector}}}{A_c \dot{q}_I} \end{aligned}$$

Since  $\frac{\dot{C}_{\text{collector}}}{A_c \dot{q}_I}$  tends to be constant with respect to  $T_{\text{Hav}}$ , we will assume that it is constant with respect to  $T_{\text{Hav}}$ . Then minimum  $\frac{\dot{C}_{\text{collector}}}{\dot{W}_{\max}}$  corresponds to maximum  $\left[1 - \left(\frac{T_H}{T_{\max}}\right)^4\right] \left(\frac{T_H - T_o}{T_H}\right)$  with  $T_{\max} = 10,500^{\circ}\text{R}$ .

Let  $y = \frac{T_H}{T_{\max}}$  and  $a = \frac{T_o}{T_{\max}}$

We assume  $T_o = 525^\circ\text{R}$  then  $a = 1/20$ .

$$\text{So } \left[1 - \left(\frac{T_H}{T_{\max}}\right)^4\right] \left(\frac{T_H - T_o}{T_H}\right) = 1 - y^4 - a/y + ay^3$$

Now we maximize  $(1 - y^4 - a/y + ay^3)$

$$\text{So } -4y^3 + a/y^2 + 3ay^2 = 0$$

$$4y^3 = a(3y^2 + 1/y^2)$$

$$\text{For } y = 1 \quad 4 > [1/20] \times (3+1)$$

$$y = 1/2 \quad 1/2 > [1/20] \times (3/4 + 4)$$

$$y = 1/3 \quad 4/27 < [1/20] \times (1/3 + 9)$$

Therefore  $1/3 < y < 1/2$ ; i.e.  $3,500^\circ\text{R} < T_{\text{opt}} < 5,250^\circ\text{R}$ .

Using trial and error methods we find  $y = 0.4240569$ , then  $T_{\text{opt}} = 4452.6^\circ\text{R}$ . This result is for the ideal case where the plant capital costs are considered to be independent of the boiler temperature, and where the effectiveness is taken to be constant over the temperature range  $4,400^\circ\text{R}$  to  $4,500^\circ\text{R}$ .<sup>\*</sup> In actual cases where capital costs depend on temperature, which dictates lower optimum boiler temperatures, let us find the optimum value of  $T_{\text{Hav}}$  at which  $10^{10}$  Btu/hr of heat enters the boiler of a power plant. Suppose that power is produced at an effectiveness of 75% and sold at  $2.85 \times 10^{-6}$  \$/Btu while the overall plant cost is, for simplicity, given by

---

<sup>\*</sup>The principle of constant effectiveness was demonstrated for the Rankine cycle over the range  $1000$ - $1500^\circ\text{R}$ , but the results would remain essentially unchanged for the range  $4400$ - $4500^\circ\text{R}$  providing, of course, that material costs were to be "irrelevant" as assumed--such a high temperature cycle then becoming "economically feasible."



$$\text{Cost} = \dot{K}_p (T_{\text{Hav}} - T_o) + \dot{C} \quad (3-65)$$

Where

$$T_{\text{Hav}} \geq T_o$$

$$\dot{K}_p = 5.25 \text{ \$/hr-}^{\circ}\text{R, a constant in this simplified example}$$

$$T_o = 75^{\circ}\text{R}$$

$\dot{C}$  = remaining cost of plant (construction and operation), taken to be constant with respect to  $T_{\text{Hav}}$ .

Because the profit ( $\dot{P}$ ) is equal to income minus cost, we have

$$\dot{P} = c_e \dot{W} - \dot{K}_p (T_{\text{Hav}} - T_o) - \dot{C} \quad (3-66)$$

Where

$c_e$  = the price at which power is sold ( $c_e = 2.85 \times 10^{-6}$  \$/Btu for this simplified analysis)

From equation (3-34) we have

$$\dot{W} = e \dot{Q}_H \left( 1 - \frac{T_o}{T_{\text{Hav}}} \right)$$

Therefore

$$\dot{P} = c_e \dot{Q}_H e \left( 1 - \frac{T_o}{T_{\text{Hav}}} \right) - \dot{K}_p T_{\text{Hav}} + \dot{K}_p T_o - \dot{C} \quad (3-67)$$

$$-\dot{P} = \dot{K}_p T_{\text{Hav}} + c_e e \dot{Q}_H \cdot T_o / T_{\text{Hav}} + \dot{C}'$$

Where

$$\dot{C}' = \dot{C} - c_e e \dot{Q}_H - \dot{K}_p T_o$$

From the principle of constant effectiveness,  $e = \text{constant}$  so that equation (3-67) is of the form  $y = Ax + B/x + C$  and the optimum value of  $x$  is  $\sqrt{B/A}$ . Therefore

$$T_{\text{Hav,opt}} = \sqrt{c_e e \dot{Q}_H T_o / \dot{K}_p}$$

This result is valid only for the given condition,  $T_{\text{Hav}} \geq T_o$ . From the given values, we have

$$T_{\text{Hav,opt}} = 1466.75^\circ\text{R} = 1006.75^\circ\text{F}$$

Thus a linear relationship between plant capital cost and boiler steam temperature dictates a temperature considerably lower than the optimum  $4452.6^\circ\text{R}$  found earlier. In actual power plants  $\dot{K}_p$  will depend on  $T_{\text{Hav}}$ . However, we may conclude here that it is only the material problems (turbine blade erosion at high temperatures, etc.) that cause operation at temperatures less than the optimum temperature ( $4452.6^\circ\text{R}$ ).

In view of this conclusion from energy availability methods, it would appear that the best procedure for utilizing the latest technology in the design of a solar power system would be to utilize a high temperature power cycle which is widely used and developed thus allowing us to reach the highest temperatures used in today's conventional power plants. We thereby select the Rankine power cycle for our study.

## CHAPTER IV

### SPECIFICATIONS

For the goal of solar energy utilization, it appears that the most sought after products are mechanical and electrical power. A number of technically successful solar power installations have been reported in the last decade, but none would be considered economically satisfactory. Admittedly, the cost of energy from a solar power system need not be as low as for a central power station because the solar system may be utilized in remote areas such as the Southwest part of the United States. But even in such areas it is limited to what the consumer is prepared to pay for solar power. If the price is too high he may prefer to go without, or to utilize some other source. We will consider the present state of art with special reference to the solar power system.

In the last decade, many technical reports discussed relatively small solar power systems, but the construction of such solar power plants would enable the engineers to "look ahead" and evaluate the potential of a solar power system. The terrestrial solar power plant should be regarded as a model for the future solar power plant. Figure 19 is a block diagram of a solar power plant operating on a Rankine cycle.

Let us now set up some specifications for our solar power plant based on the previous energy availability analysis.

1. High temperature--The temperature is lowered only because of

the dependence of material cost on temperature.

2. Rankine cycle--Most economic, high temperature power cycle based on current technology (steam temperatures of the boiler as high as  $1050^{\circ}$  at 3650 psia).<sup>60</sup>

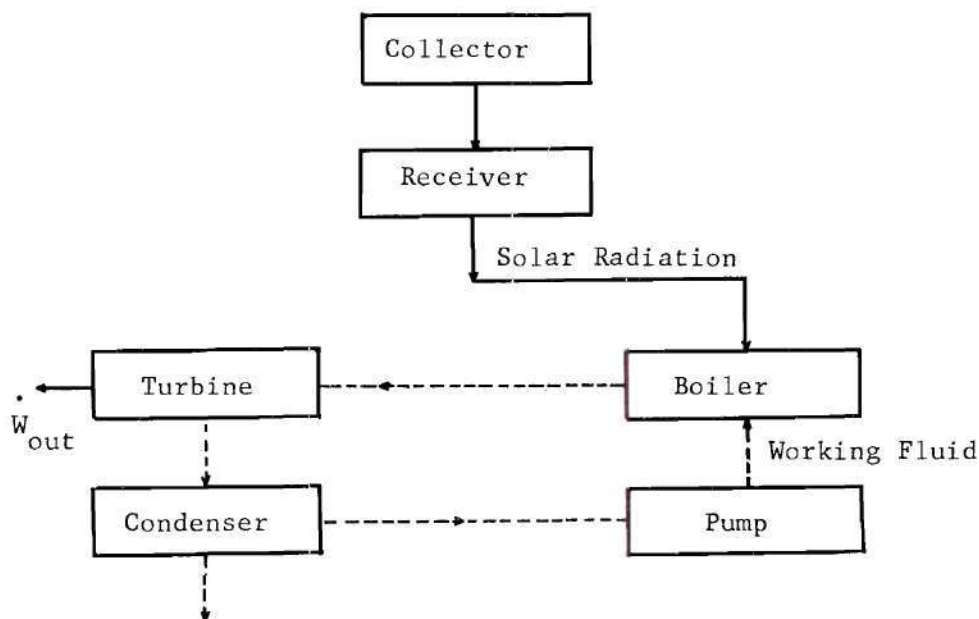


Figure 19. The Block Diagram of a Simplified Solar Power Plant Using the Rankine Cycle

#### Optical System (Solar Collector)

There are two types of solar power plants with and without concentrators. More promising from the power generation and economic standpoint are plants with focusing solar radiation collectors. In this case, collecting and focusing the solar radiation is the most important design consideration of the solar power plant. The optical system of the solar

power plant must be quite large and represent a considerable proportion of the capital investment. Therefore, the efficiency and the economy of the solar power system depends to a large extent on the efficiency and cost of the optical system.

A solar power plant may consist of individual units, each unit with an independent optical system. These units are combined to form a power plant to produce energy.

The function of the optical system of a solar power plant is similar to the dam and reservoir of a modern hydroelectric power station whose performance depends on the height of the dam and the charge of the reservoir. The performance of a solar power plant depends upon the size and concentration capacity of the optical system and the material employed.

The parameter of any reflecting solar optical system are determined first by the intensity of the reflected radiant flux and second by the mean geometric and maximum concentration of energy at the receiver. These parameters depend on the size of the reflecting surface and its type. The higher the concentration ratio, the higher the temperature at the receiver that will be needed; the greater the thermal efficiency of the solar power plant will be obtained.

The optical system of the solar power plant must operate in the open and be able to withstand weathering and wind load. Thus the problems of protecting the elements of the system and designing for adequate strength and rigidity are very important. Also, we conclude from the energy availability analysis that the need for as high a temperature as possible dictates a maximum concentration ratio for the optical system at a reasonable cost, such as \$7 50/ft<sup>2</sup>. Because the Fresnel lens has con-



struction problems (flat-plate type, the concentration ratio is too low, etc.), its cost is too expensive. Thus our considerations lead us to use a parabola of revolution mirror because of its high concentration ratio, ease of handling, and lower expense.

From the above discussion we choose the type of collector to be a parabola of revolution mirror. For this kind of mirror, the diameter limits should be between seven and three feet, because the collector must be easy to handle and clean.

For cleaning the reflecting surface, the mirror will probably be transported to a central cleaning system, which can employ blowing, washing with water, and subsequent mechanical wiping of the reflecting surface.

#### Concentration Ratio

A useful parameter descriptive of a focusing system is the concentration ratio. High concentration ratios are associated with high temperatures and precise optics. There are practical limits on the concentration ratio with the requirement of good collector performance. The relationship between concentration ratio and receiver temperature resulting in solar energy collection can be derived from a simple energy balance which is tabulated in Table 6.

Table 6 is derived from Equations (3-58) and (3-60) and the following relationship for the effectiveness  $e_c$ .

$$e_c = \eta_c \frac{T_H - T_o}{T_H} \quad (4-1)$$

Table 6. The Relationship among the Concentration Ratio, Receiver Temperatures,  $\eta_c$ , and Collector Effectiveness  $e_c$  (Neglecting Losses from Convection and Conduction).

$q_I$ Btu/ft <sup>2</sup> hr	X	$T_H$ °R	$T_O$ °R	$q_n$ Btu/ft <sup>2</sup> hr	$\eta_c$	$e_c$
275.8	100	1460	535	19792.07	71.76%	0.457
275.8	100	1960	535	2284.98	8.28%	0.061
275.8	100	2460	535	1431.99	5.20%	0.041
275.8	1000	1460	535	268013.07	97.18%	0.621
275.8	1000	1960	535	250504.98	90.83%	0.653
275.8	1000	2460	535	249651.98	90.52%	0.709
275.8	10000	1460	535	2750222.07	99.72%	0.636
275.8	10000	1960	535	2732704.98	99.08%	0.710
275.8	10000	2460	535	2731851.99	99.05%	0.766

Since from our previous energy availability analysis, higher temperatures are more desirable (up to the optimum temperature of 4452.6°R), it is seen from Table 6 that the collector effectiveness  $e_c$  is a much better measure of collector performance than the conventional efficiency  $\eta_c$ .

To achieve high temperatures at high collector effectiveness, we should use as high a concentration ratio as possible (compatible with a reasonable cost of the mirror).

#### Receiver

The receiver is designed to absorb the focused radiant energy.

The absorption and primary transformation of radiant energy should result in minimum energy losses, i.e. the absorption coefficient should be maximum. The type and design of the receiver are determined by the selected energy conversion system, the design parameter of the focusing optical system of the solar power plant.

An important component of any turbine solar power plant is the solar boiler in which the working fluid from the condenser is heated and vaporized. From discussion in the previous chapter, it is evident that we need as high a temperature for our plant as possible, dictating use of a maximum concentration ratio. In order to absorb this energy without burning up the receiver, we choose a reflective black-body cavity-type absorber--it can absorb most of the radiant heat from the sun and the cavity is small corresponding to the high concentration ratio that we need. Stephen and Hire<sup>61</sup> have considered the following absorber boiler configurations: a cavity type, a flat-plate type, and a hemisphere type. They concluded that, even for very favorable selective absorber surfaces, the cavity-type boiler is superior to the others. This conclusion supports our choice of the cavity-type boiler. For convenience, we will call such a cavity type boiler a solar well. One type of solar well is shown in Figure 20 and another in Figure 21.

The aperture of the solar well is very small; it is less than one inch in diameter (0.6 inch for  $X = 10,000$ , five feet diameter mirror--see Appendix A). The sun rays are collected by the solar collector then reflected into the aperture to produce a high temperature in the solar well.

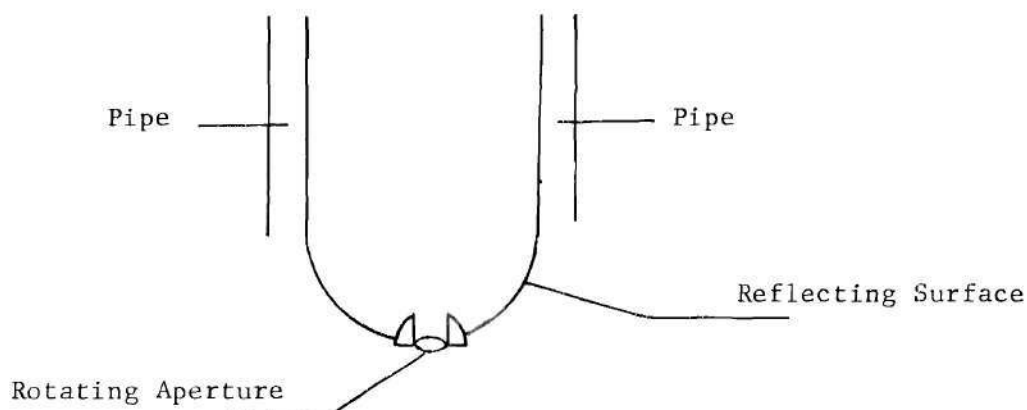


Figure 20. Proposed Solar Well for Low Value of  $\dot{q}_{\text{net}}$   
(Transfer Area  $\cong 10 \text{ ft}^2$ )

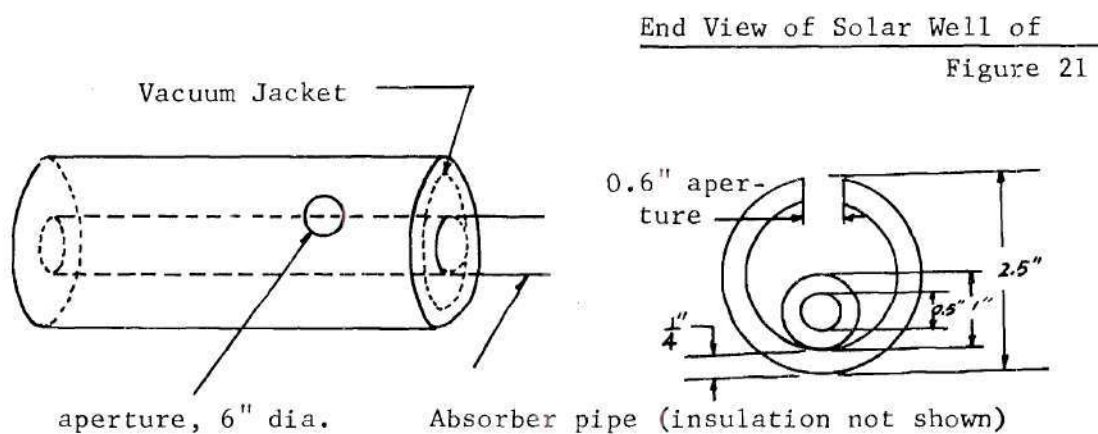


Figure 21. Proposed Solar Well for High Values of  $\dot{q}_{\text{net}}$   
(Requiring a Transfer Area  $\approx 0.1 \text{ ft}^2$ )

If the aperture does not rotate, then the concentrated rays may strike outside the aperture as shown in Figure 22 causing extensive damage (melting the solar well, etc.). Thus some rotation of the aperture is required, or an unnecessarily large aperture would be needed, giving an



unnecessarily low effectiveness (as calculated on page 75). The solar well as shown in Figure 21 rotates around the receiver pipe so that the aperture follows the focal point (as suggested by Manhefsky<sup>62</sup>).

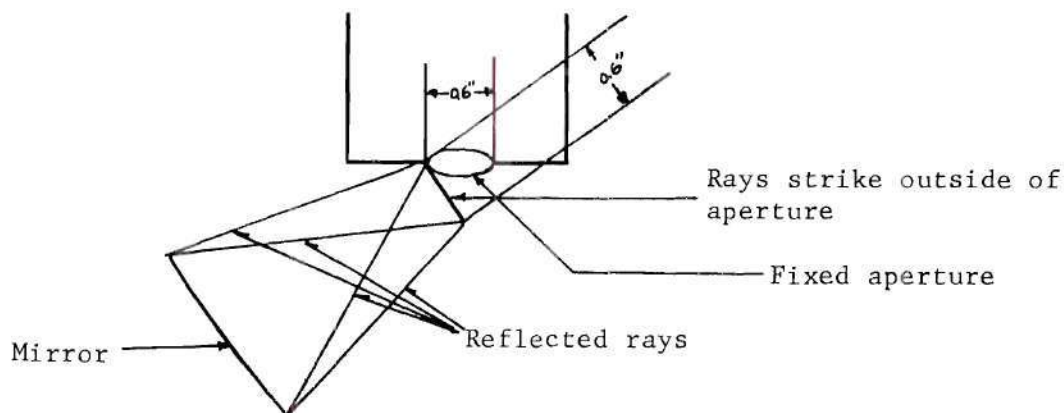


Figure 22. Solar Well with a Fixed Aperture Showing How Concentrated Rays May Strike Outside the Aperture, Causing Extensive Damage (See Appendix for Calculation of the 0.6 Inch Focus Diameter)

#### Tracking System

In view of the diurnal and annual motions of the sun, the collection elements of a solar power plant can not remain stationary. There are three possible variant combinations:

1. Moving the collector with a stationary receiver,
2. Moving the solar well with a stationary collector,
3. Simultaneously moving the solar well and the collector.

The motion of the sun relative to the polar axis takes place at the rate of one revolution per day--1/1440 rpm (equatorial rate). In an equatorial kinematic system the collector must rotate about some axis parallel



to the polar axis at a uniform equatorial rate. In an azimuthal-zenithal system this uniform rate is ensured by superimposing two nonuniform rotations, vertical and horizontal. In both systems the rates of displacement are small and tracking accuracy can be ensured only by an automatic control device.

When the tracking system is switched on periodically the installation always operates in the variable angle defocusing mode and this reduces its efficiency. However, small interruptions do not lead to significant energy losses. If the system consists of a large number of independent solar collectors with individual devices, they should be standardized, thus reducing the capital cost. One proposed tracking system is shown in Figure 23.

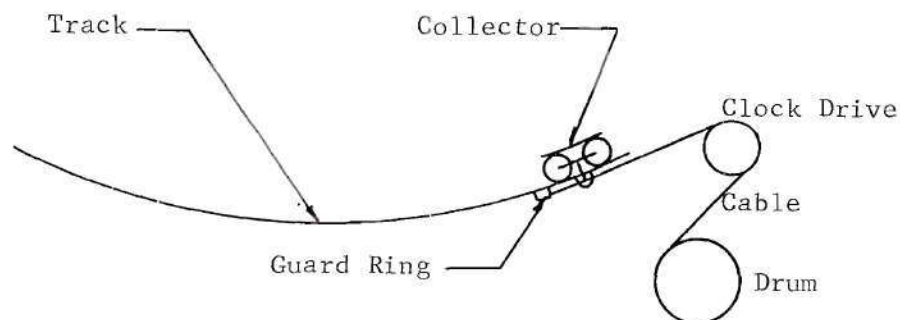


Figure 23. Proposed Tracking System

The solar collector moves from right to left when the sun rises in the morning and sets in the evening. Each sun's ray focuses to within a specified distance of the focusing point. If the aperture is not rotating, then the sun's rays can not reach into the aperture as shown in Figure 22.

Thus some rotating of the aperture is required.

The tracking system uses a cable car on tracks pulled by a clock drive; the rotating aperture is also run by a cable from the same clock drive. The cable is guided by the guard ring, as shown in Figure 23. A couple of alternative tracking systems are shown in Appendix B. These alternative systems show promise of being considerably less expensive.

#### Control System

We must set up an automatic control mechanism for the tracking system to avoid the heat striking the edges of the aperture and burning up the material. The clock drive unit will only be able to keep the rays to within some specified distance of the rotating aperture--say within a window of three inches diameter. The automatic control system must function within this window to keep the rays focused within the rotating aperture.

A complicated control system may also include start up, shut down, normal operation, etc. An automatic start up would mean automatic "searching" for sun light corresponding to the orientation of all the reflecting elements of the collector, and guidance of the receiver and the converter into the operating mode. Under operating conditions the automatic system would ensure continuous tracking of the sun and would initiate special maneuvers to exclude the possibility of the solar well overheating when the radiation intensity exceeds the design limit. When the sun is briefly obscured, the tracking system would either be switched off or onto continuous "tracking" of the sun. When the sun appeared, the solar power plant

would return to operation, as in the morning. If the weather is not clear enough for collecting the sun rays, then the tracking system will automatically shut down.

#### Summary of Specifications

From the above discussion, we will summarize the specifications of our solar power plant.

1. The solar collector (parabola of revolution mirror) of high effectiveness at reasonable cost ( $\$7.50/\text{ft}^2$ ).
2. The diameter of the solar collector limited to between seven feet and five feet.
3. A solar well with an aperture of about 0.6 inch diameter.
4. A tracking system with a clock drive that makes the sun's ray focus to within a specified distance of the focusing point, requiring a rotating aperture.
5. An automatic control system operating within the clock-drive window to keep the sun's rays focused within the rotating aperture.
6. A focal length of about five feet.

A sketch of a system which might meet these specifications is shown in Figure 24.

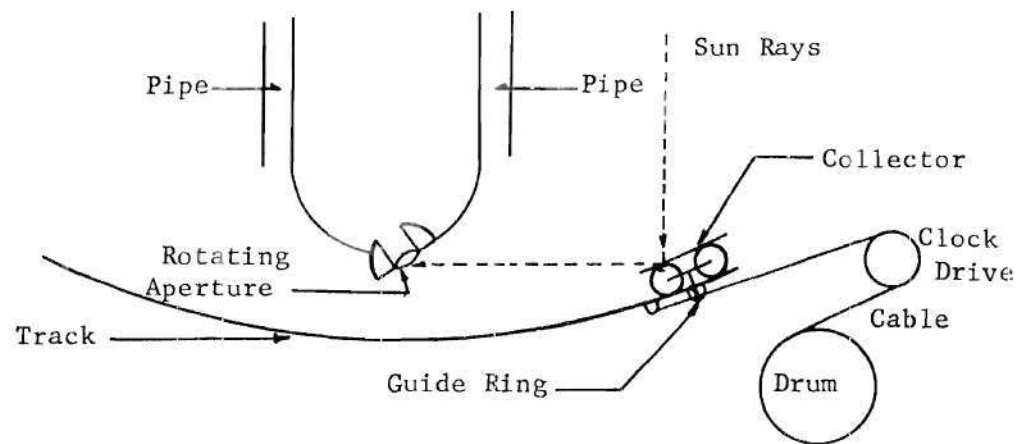


Figure 24. The Diagram of the Focusing Elements of a Solar Power Plant

## CHAPTER V

## DESIGN CONSIDERATIONS

Now we will consider a 1,000 megawatt Rankine cycle solar power plant. As envisioned in this design, the collector concentrates solar radiation to heat a working fluid (mercury, aluminum, etc.) to a temperature considerably higher than cycle temperature. This fluid is pumped to a storage tank where it is withdrawn as needed by the steam generator. The steam generator uses this fluid to supply the heat needed to generate and superheat steam. The steam turns a conventional turbo-generator for electric power production. The turbine exhaust steam is condensed and returned to the generator through a feed water pump. Figure 25 is the schematic of this solar power plant.

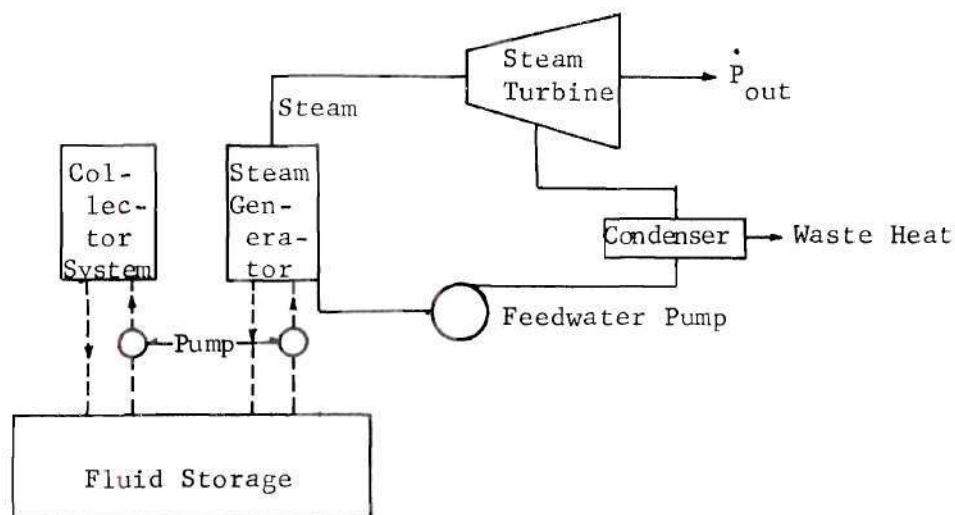


Figure 25. Schematic of Solar Power Plant



Now we will consider a more detailed design for this power plant.

### Design Parameters

The collector is an energy supply system for delivering heat over a wide range of temperatures, and thus meets the requirements of temperature and energy flux of a variety of power generation processes. The solar collector supplying energy for conversion to electricity is a unique variety of heat exchanger which performs the function of energy transfer from a distant source of radiation to a working fluid. In our selection, the focusing collector system is concentrated by an optical system (parabola of revolution mirror) to increase the radiant flux prior to striking the absorption area where some thermal losses will occur.

The essential parts of the focusing collector, useful for power generation, are shown in Figure 24. They are the reflecting surface and its supporting shell, and the receiver (solar well) with its energy-absorbing surface. This system must track the sun and properly focus the solar radiation.

There are many possible configurations of reflectors and receivers with the selection of a combination dependent on the conversion process and the temperature at which it must operate. The theoretical parabola of a revolution mirror will focus parallel incident radiation to a point. The receiver in our power plant is a solar well (cavity type). This receiver absorbs the solar radiation and thus should have a high absorptivity for radiation in the solar energy wavelength range (0.3 to 2.5 microns).

The performance of a focusing collector of any particular design operating under specified conditions may be described in part by an



energy balance.<sup>63</sup>

$$\text{ENERGY IN} = \text{HEAT LOSS} + \text{USEFUL ENERGY}$$

$$\dot{q}_{\text{net}} A_c \gamma \rho \alpha \tau = UA_j (T_{\text{Hav}} - T_a) + \sigma \epsilon \bar{F}_{1-2} A_a (T_{\text{Hav}}^4 - T_a^4) + \dot{q}_{\text{net}} A \quad (5-1)$$

where

$A$  = the surface area of solar well,

$A_j$  = the surface area of jacket; it is equal to  $4A$ ,

$U$  = heat transfer coefficient for vacuum jacket; it is equal to  $0.25 \text{ Btu/hr-ft}^2 \cdot ^\circ\text{F}$ ,

$\bar{F}_{1-2}$  = shape factor.

#### Optical and Thermal Losses

In order that the useful energy can be as large as possible, the optical and thermal losses from the system should be minimized. The product  $\gamma r$  represents the portion of the solar energy incident on the reflector surface which is intercepted by the receiver.<sup>63</sup> The reflectivity of the surface  $r$  depends on the material of which the reflector is made and surface smoothness; it is in the range of 0.7 to 0.9.

The intercept factor ( $\gamma$ ) represents the fraction of the total specular reflected radiation which reaches the absorbing surface. It is unity if the energy absorbing surface is sufficiently large to intercept all beams of reflected radiation and it may range downward to 0.5 in some rather crude focusing collectors.<sup>64</sup> The intercept factor is a function

of the receiver size for a given reflector, and also depends on the accuracy of the solar alignment of the collector. Performance optimization studies on various collectors indicate that values of  $\gamma$  in the range of 0.9 to 1 lead to maximum useful heat delivery in the most practical temperature ranges. Figure 26 shows the relationship between the average angular error (of a parabola of revolution mirror) and concentration ratio for various  $\gamma$  in the range of 0.92 to 0.98.

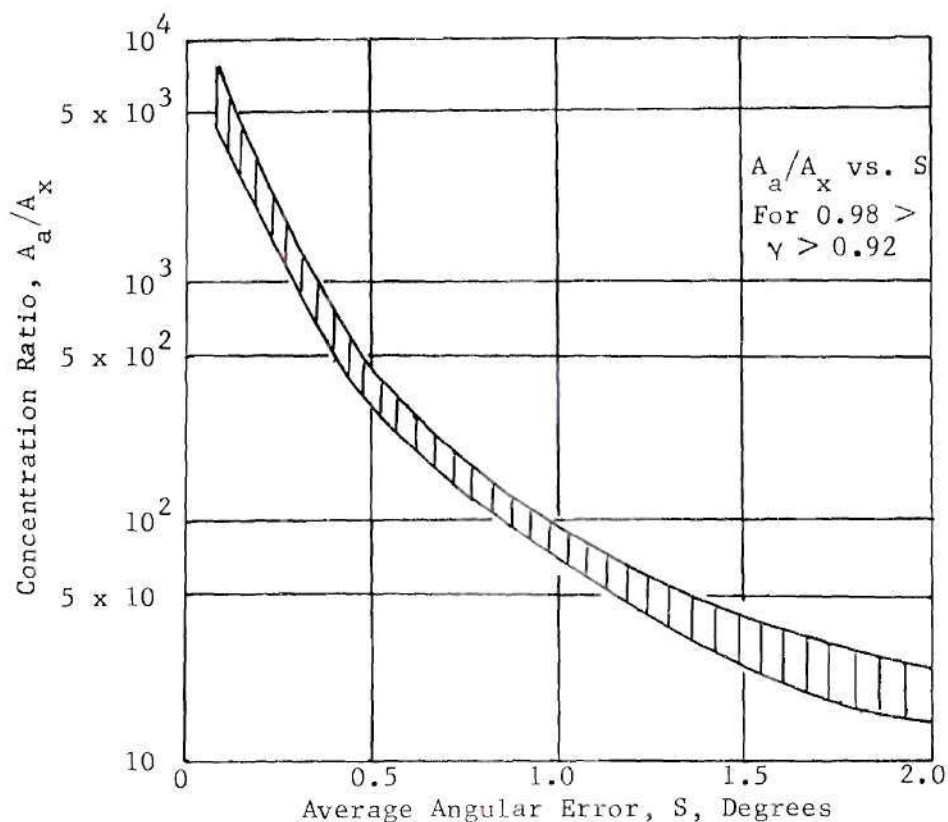


Figure 26. The Relationship between the Average Reflector Angular Error (of a Parabola of Revolution Mirror) and Concentration Ratio for Various  $\gamma$  in the Range of 0.92 to 0.98<sup>63</sup>

The factors  $\tau$  and  $\alpha$  in the solar radiation terms of the energy balance represent the fractions of radiation transmitted by any transparent insulating surface used to reduce convection and radiation losses from the solar well, and the fraction of the energy incident on the solar well which is absorbed.

The product of  $\tau_r \alpha$ , which may be termed the overall optical efficiency of the reflector-receiver system, limits the total energy available to the solar well (receiver). The range of values for this product will be between 0.9 and 0.5.

Heat losses by conduction, convection and radiation from the receiver may make use of the surface and its surroundings, the heat transfer coefficients, and emissivity of the surface to permit determination of these quantities by established methods.<sup>65</sup> If a transparent surface is employed to minimize the thermal loss, the heat transfer coefficient  $U$  must be an overall term applicable to the heat flow; the radiation transfer expression must then be modified to apply to the temperature difference between the receiving surface and the transparent cover (assume the cover is opaque to the thermal radiation) and from the cover to the atmosphere. Radiation losses may be minimized by use of a surface with selective radiation properties with low values of emissivity while retaining high values of absorptivity for solar radiation. Table 7<sup>64</sup> presents some  $\alpha$  and  $\epsilon$  data for surfaces for solar energy applications.

Table 7. Some  $\alpha$  and  $\epsilon$  Data for Surfaces for Solar Energy Applications

Surface	$\alpha$ for solar radiation	$\epsilon$ , at temp less than 100°C	Approx. upper temp. range of application
CuO on Al	0.85	0.11	200°C
Nickel black on galvanized iron	0.89	0.12	
Ebonol-C on copper	0.91	0.16	400°C
Stainless Steel, 16% cr., heated 3 or at 600°C	0.75	0.1	
Aluminum treated with $\text{KMnO}_4$	0.80	0.35	
Commercial flat black paints	0.93	0.93	
Platinum black	0.95	0.91	900°C
$\text{Co}_3\text{O}_4$ on Pt.		0.13	1000°C

### Collector Effectiveness

Collector effectiveness is the ratio of the useful available energy delivered to the total direct solar radiation incident on the reflector surface. From Table 6 we know that the higher collector concentration ratio we have the higher collector effectiveness we get. But Table 6 is for the ideal case; it does not consider the reflectivity, emissivity, shape factor or absorptivity, etc. For our design considerations we chose the concentration ratio as 10,000, and  $\tau\alpha$  equal to 0.95. Then from Equation (5-1) and the definition of collector effectiveness we get

$$\dot{q}_I \delta r \alpha T_c = U A_j (T_{Hav} - T_a) + \sigma \epsilon \bar{\epsilon}_{1-2} A_a (T_{Hav}^4 - T_a^4) + \dot{q}_{net} A$$

where  $A_j = 4A$ ; the value of  $A$  is  $1.815 \times 10^5 \text{ ft}^2$ ; the value of  $A_c$  is  $3.257 \times 10^7 \text{ ft}^2$  (those values come from page 78 and page 79).

Therefore

$$\begin{aligned} \eta &= \frac{\dot{q}_{net} A}{\dot{q}_I \delta r \alpha T_c} \\ &= \frac{\dot{q}_I \delta r \alpha T_c - 4A(T_{Hav} - T_a) - \sigma \epsilon \bar{\epsilon}_{1-2} A_a (T_{Hav}^4 - T_a^4)}{\dot{q}_I \delta r \alpha T_c} \\ &= 1 - \frac{4U(T_{Hav} - T_a)}{\dot{q}_I \delta r \alpha \frac{A_c}{A}} - \frac{\sigma \epsilon \bar{\epsilon}_{1-2} A_a}{\dot{q}_I \delta r \alpha T_c} \\ &= \eta \left( \frac{T_s - T_a}{T_s} \right) \end{aligned}$$

From our energy availability analysis and specifications we know  $T_{Hav} = 2000^\circ\text{F}$ ,  $T_s = 1050^\circ\text{F}$ ,  $U = 0.25 \text{ Btu/hr-ft}^2\text{-}^\circ\text{F}$ ,  $\sigma = 0.1714 \times 10^{-8} \text{ Btu/hr-ft}^2\text{-}^\circ\text{R}^4$ ,  $\epsilon = 1$  (black-body solar well),  $A_a/A_c = 1/X = 0.0001$  and  $\bar{\epsilon}_{1-2} = 0.8$ ,  $T_a = 75^\circ\text{F}$  then

$$\eta_c = \left( 1 - \frac{1925}{4.713 \times 10^4} - \frac{0.13712 \times 0.3654}{1.94} \right) \times \% = 93.6\%$$

$$\eta_c = 93.6\% \times \left( \frac{1510 - 535}{1510} \right) = 60.47\%$$

The better the reflector design quality and the higher the concentration ratio, the higher the collector effectiveness.



## Materials

The discussion of material properties is conveniently treated in three sections: 1. Reflecting radiation, 2. Absorbing radiation, and 3. Structural parts for supporting and moving the reflector surface and receiver.

1. Reflecting surfaces for focusing collectors should have high reflectivity. Three types of materials have been proposed for use to provide reflecting surfaces.<sup>63</sup> These are back-silvered glass, vacuum-metallized plastic film and relatively thick metal sheets.

2. Energy absorbing surfaces for focusing collectors must have high absorptivity for solar radiation. It is also desirable that they should be selective, i.e., have low emissivity. Table 7 shows  $\alpha$  and  $\epsilon$  data for several surfaces with an indication of an upper limit of temperature of operation of the surface where available information permits estimation of the limit.<sup>63</sup>

3. The shell material provides the necessary support for the reflective surface and maintains the desired level of reflector contour accuracy. Shells may be plastics, metals or other materials that can be shaped to the required simple or compound curvature and applied to the reflective surface. Several plastic materials have already been used in the laboratory. Steel and aluminum appear to be the principal materials for the supporting components.<sup>63</sup>

## Energy Storage

In spite of the stability of the annual supply of solar radiation, it is subject to considerable fluctuations over shorter periods. Accordingly, the energy of a solar power plant can be utilized in two principal



ways: 1. the energy produced by the plant is stored continuously in water storage; or, 2. the energy produced by the solar power plant is released periodically (intermittently) into the grids. Besides those two alternatives, other energy storage systems are flywheels, batteries, fuel cells and storage of the intermediate hot fluid, etc.

### Plant Design

The design of a solar power plant requires knowledge of the necessary heat delivery rate and the estimation of the probable range of useful operating temperatures. With knowledge of the solar radiation intensities available in the location of the application, the optimum concentration ratio, and the reflector accuracy, the temperatures within the useful range may be determined by the use of solar collector optical relationships and energy balances for the solar collector system. After that we can set up our power plant with conventional power plant knowledge.

Now we will consider these things and then set up the general design parameters for our solar power plant.

1. We assume the average daytime (8 hours/day) power production rate in our study is  $10^9$  watts.
2. The operating maximum of the superheated steam is  $1050^{\circ}\text{F}$ .
3. The solar radiation intensity in the Arizona desert is  $275.8 \text{ Btu/hr-ft}^2$ , because we choose our solar power plant location in the Arizona desert. This value is a mean value (i.e.,  $\dot{q}_I = 275.8 \text{ Btu/hr-ft}^2$ ).<sup>66</sup>
4. The collector diameter is between 7 feet and 3 feet, so let us select 5 feet.
5.  $X = 10,000$ , from Table 6 and material considerations, we selected the temperature of the solar well as  $2000^{\circ}\text{F}$ .

6.  $\epsilon = 1$  for black-body cavity type receiver.
7.  $\tau_r \tau_d = 0.95$ .
8. The inside diameter of the absorbing pipe is 1/2 inch.
9. The total power plant efficiency is 40%.

First we want to find the collector area:

$$\eta_{\text{tot}} = \frac{\dot{W}}{\dot{q}_I A_c \tau_r \tau_d}$$

where  $q_I = 0.0808 \text{ kw/ft}^2$

$$\begin{aligned} &= \frac{\dot{W}}{\eta_{\text{tot}} \dot{q}_I \tau_r \tau_d} \\ &= \frac{10^6 \text{ kw}}{0.0808 \times 0.4 \times 0.95} \\ &= 3.257 \times 10^7 \text{ ft}^2 \end{aligned}$$

So, the total collector area is  $3.257 \times 10^7$  square feet. Dividing by a  $\frac{\pi(5)^2}{4}$ , the number of collectors that we need is about  $1.66 \times 10^6$ .

Second, we want to discover how much piping is needed for the solar wells. After Kreith,<sup>67</sup>

$$\begin{aligned} \dot{q}_{\text{net}} &= h (T_{\text{Hav}} - T_s) \\ &= 50(2000 - 1050) \\ &= 4.75 \times 10^4 \text{ BTU/hr} - \text{ft}^2 \\ &= 14.91 \text{ kw/ft}^2 \end{aligned}$$

$$\dot{W} = \dot{q}_{\text{net}} A \eta_p$$

where  $\eta_p$  = steam power plant efficiency

$$A = \frac{\dot{W}}{\dot{q}_{\text{net}} \eta_p} = \frac{10^6 \text{ kw}}{14.91 \times 0.437} = 1.815 \times 10^5 \text{ ft}^2$$

$$A = \pi D L$$

$$L = \frac{A}{\pi D} = \frac{1.815 \times 10^5}{\pi \frac{1}{2 \times 12}} = 1.38 \times 10^6 \text{ ft}$$

The length of piping and vacuum jacketing needed for the solar wells is about  $1.38 \times 10^6$  feet (these pipes are one inch in outside diameter and half inch in inside diameter).

For the total power plant we need  $3.257 \times 10^7$  square feet of collector surface for 1,000 megawatt average daytime power output and  $1.38 \times 10^6$  feet of insulated pipe for the solar wells. For delivery of the steam from the solar wells to the central power plant, figure on an average of 5 feet of one inch insulated pipe per collector plus 5,000 feet of one foot diameter manifold piping. For  $1.66 \times 10^6$  collectors, this requires about  $8.3 \times 10^6$  feet of one inch pipe. Table 8 shows the design parameters for a 1,000 megawatt average daytime output power plant.

Table 8. The Design Parameter for a 1,000 Megawatt Output Rankine Power Cycle

Solar Collector	
Mean daily insolation	275.8 Btu/ft <sup>2</sup> hr
Reflector type	parabola of revolution mirror
Collector area	3.257 x 10 <sup>7</sup> ft <sup>2</sup>
Collector efficiency	93.66%
Collector effectiveness	60.47%
Heat Source	
Working fluid	steam
Solar well temperature	2000 <sup>o</sup> F
Energy Storage	water storage
Receiver Type	solar well (black-body cavity-type)
Steam Generator	
Water inlet condition	at 3650 psi, 101 <sup>o</sup> F
Exit steam temperature	1050 <sup>o</sup> F
Exit steam pressure	3650 psi
Rankine Cycle	
Net power output	1000 megawatt
Net cycle efficiency	40%

Cost Estimate

The first thing to be discussed is the conventional power plant

cost. Assuming the total power plant cost is  $\$10^8$  and the plant life is 25 years.

$$\dot{C}_{\text{plant}} = \frac{10^8 \times 100 \text{¢}}{25 \times 24 \times 365} \times (F)$$

where (F) includes interest, maintenance, etc. Let (F) equal 3.75.

$$\begin{aligned} \dot{C}_{\text{plant}} &= \frac{10^8 \times 3.75 \text{¢}}{25 \times 24 \times 365} \\ &= 1.2712 \times 10^5 \text{ ¢/hr} \end{aligned}$$

The average daytime power output is 1,000 megawatt.

$$\frac{\dot{C}_{\text{plant}}}{\dot{W}} = \frac{1.2712 \times 10^5 \text{ ¢}}{10^6 \text{ kw}} = 1.2712 \times 10^{-1} \text{ ¢/kw-hr}$$

Second we will discuss the fuel cost for a conventional power plant. Assuming the typical fuel cost for oil, coal, gas, etc. is  $90\text{¢}/10^6 \text{ Btu}$ , if the fuel efficiency is 45%, then fuel cost is  $\frac{90}{10^6} \times 3415 \times \frac{1}{0.45} = 0.683\text{¢/kw-hr}$ . So, the fuel cost for a conventional power plant is about five times the plant cost.

The cost of the solar power plant should be compared to the cost for a conventional power plant of similar capacity which would meet similar needs. At this time, the cost of solar power plants has not been determined but a major item of capital expense is the collector system.

Limited data on the cost of focusing solar collectors of several



types have been published. The overall collector cost, including the reflector and its supporting structure, can be bracketed between  $\$2/\text{ft}^2$  and  $\$10/\text{ft}^2$  of surface. It should be possible to manufacture a parabola of revolution mirror having shells of various molded or cast plastic materials, lined with metal foil or metallized plastic film for  $\$1/\text{ft}^2$  to  $\$3/\text{ft}^2$ .

Now we will estimate the cost for the solar power plant very roughly.

1. The cost of the collector system which includes reflectors and its supporting structures is 7.5 dollars per square foot. Then the cost for the total collector system is  $2.44275 \times 10^8$  dollars.

2. The cost of water storage and additional generators, water pumps and turbines is  $4 \times 10^7$  dollars.

3. If the cost of the solar well is  $\$10/\text{ft}$ , then the total cost of this item is  $1.38 \times 10^7$  dollars.

4. For delivery of the steam from solar well to the central power plant turbines, figured on the average of five feet of one inch diameter insulated pipe per collector plus 5,000 feet of one foot diameter manifold piping. For  $1.66 \times 10^6$  collectors, we obtain  $8.3 \times 10^6$  feet of one inch pipe. At a cost of  $\$8/\text{ft}$ , this comes to  $6.84 \times 10^7$  dollars for insulated piping to the manifold. At  $\$50/\text{ft}$  of manifold pipe, this costs  $2.5 \times 10^5$  dollars. Total cost of pipe for steam delivery to central power plant is equal  $6.865 \times 10^7$  dollars. This cost may be lowered by using an intermediate fluid (aluminum, sodium, mercury, etc.) as shown in Figure 25, thus eliminating the high pressure piping.

5. The cost of the steam power plant is  $10^8$  dollars.



6. The power output reduction from water storage is 20% (80% combined efficiency of water pumps, water turbines, and electric generator, etc.).

The total cost of the solar power plant is  $4.66725 \times 10^8$  dollars.

Using (F) at the same rate, i.e., 3.75, and same life time, then, taking the steady-state power output 1/3 of average daytime power output, therefore

$$\frac{\dot{C}_{\text{plant}}}{\dot{W}} = \frac{4.66725 \times 10^8 \times 10^2 \times 3.75}{\frac{1}{3} \times 24 \times 365 \times 25 \times 10^6 \times 0.8} = 2.1 \text{ ¢/kw-hr}$$

Comparing this value to the conventional power plant value (0.80712¢/kw-hr), we find it over twice as high as for a conventional power plant. Clearly, the cost of the collectors must be reduced, if solar power plants are to be competitive with conventional steam power plants.

## CHAPTER VI

### ANALYSIS OF RESULTS

#### Summary and Conclusion

As is noted in Chapter III, using energy availability methods, we find that the effectiveness of a power cycle is virtually constant with respect to the collector temperature, a principle which may be called the principle of constant effectiveness. For idealized situations where the capital cost of a power plant is taken to be independent of the boiler steam temperature, this principle of constant effectiveness dictates an optimum boiler temperature of  $4452.6^{\circ}\text{R}$ . Thus for any actual plant, it is strictly the dependence of material costs on temperatures which dictates lower optimum boiler temperatures. In other words, there is nothing in the laws of physics concerning collector efficiencies which points toward a lower optimum temperature than  $4452.6^{\circ}\text{R}$ .

In view of this conclusion from energy availability methods, it appeared that the best way to proceed in utilizing the latest technology in the design of a solar power system was to utilize a high temperature power cycle which is widely used and developed, thus allowing us to reach the highest temperature used in today's conventional power plants. We thereby selected the Rankine power cycle for our study. Appropriate performance specifications were found to be

- a) A solar collector (parabola of revolution mirror) of high

effectiveness (60.47%) at a reasonable cost  $\$7.5/\text{ft}^2$  or less.

b) The diameter of the solar collectors is five feet.

c) The receiver type is of the cavity type (called a solar well), the aperture diameter being about 0.6 inch.

d) A tracking system that keeps the sun's rays focused within the aperture. To aid in this, we use a rotating aperture coupled with an automatic control system.

A solar power plant which meets these specifications was then designed in general. This power plant needs  $3.275 \times 10^7$  square feet of collector surface for a 1,000 megawatt daytime output and  $8.3 \times 10^6$  feet of insulated pipe for delivery of the working fluid. The rough capital cost estimate for the entire solar power plant is  $4.35725 \times 10^8$  dollars, which leads to a power cost of 2.1¢/kw-hr--or over twice the cost of power from a conventional steam power plant. Most of this cost is due to the cost of the collector system ( $3.35575 \times 10^8$  dollars).

The major disadvantages of a solar power plant stem from the fact that the sun is available for only a few hours a day and the collecting surface must be enlarged to provide power for storage to be used during the dark hours and on overcast days. The initial cost of these collecting areas is high and considerable real estate is required. However, use of such a power generation facility is pollution free (requires no fuels for operation, etc.). Therefore, in comparing the cost of such a plant to the conventional plant cost with its accompanying fuel cost, the initial capital cost is much more acceptable. Research is required to reduce the cost of the mirrors and their supports. This cost reduction could probably

be realized by using the tracking mechanism and mirror support introduced in this thesis (Figure 21 and Appendix B) which could result in a mirror cost considerably lower than the assumed  $\$7.5/\text{ft}^2$ .

The most logical location for a large solar power plant would be in southwestern Arizona, since it has the highest percentage of sunshine in the United States and is not densely populated.

#### Recommendations

For future study we suggest the following recommendations.

1. A detailed cost study of the mirror supports and tracking mechanisms introduced in this report should be undertaken.
2. The energy storage problem should be examined: study of ways for reducing the cost of water storage, and development of other methods such as storage of a hot intermediate fluid, flywheel, fuel cells, batteries, etc. that might reduce the cost of energy storage. Also, by using solar power as a supplement to existing power sources, the energy storage requirement may be greatly reduced.
3. A suitable material for the solar well that can sustain higher temperatures should be identified.
4. An in-depth study should be undertaken to determine the most effective utilization of the higher temperatures, which are dictated by the energy availability methods used in this research.

## APPENDIX A

## THE DIAMETER OF THE ROTATING APERTURE

In view of Table 6, we chose the concentration ratio to be approximately its maximum possible value which is 10,000, as indicated by the following simplified calculations. For the diameter of the collector of five feet, we find the diameter of the rotating aperture to be 0.6 inch as follows.

From the thin lens quasi-flat mirror formula we know (see figure below)

$$\frac{d}{y} = \frac{d'}{y'}$$

where

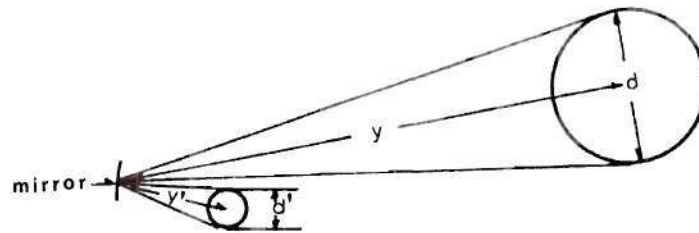
$y$  = the distance from the sun to the earth, about  $9.3 \times 10^7$  miles.

$d$  = the diameter of the sun, about  $9.3 \times 10^5$  miles.

$y'$  = the distance from the collector to the aperture (focal length), five feet.

$d'$  = the diameter of the aperture.

$d''$  = the diameter of the collector, five feet.



$$A_c = \frac{\pi d'^2}{4} = \frac{\pi y'^2}{4}$$

$$A_a = \frac{\pi d^2}{4}$$

$$x = \frac{A_c}{A_a} = \left(\frac{y'}{d'}\right)^2$$

Therefore

$$\begin{aligned} x &= \frac{y'^2}{d'^2} \\ &= \frac{y^2}{d^2} \\ &= \left(\frac{93 \times 10^6}{93 \times 10^4}\right)^2 = 10^4 \end{aligned}$$

Thus

$$10^4 = \frac{y'^2}{d'^2} = \frac{25}{d'^2}$$

Therefore

$$d' = 0.05 \text{ ft}$$

$$= 0.6 \text{ inch}$$

From this calculation we find that the diameter of the rotating aperture is 0.6 inch.



## APPENDIX B

## TWO OTHER TYPES OF THE TRACKING MECHANISM

The following figures present two alternative tracking mechanisms.

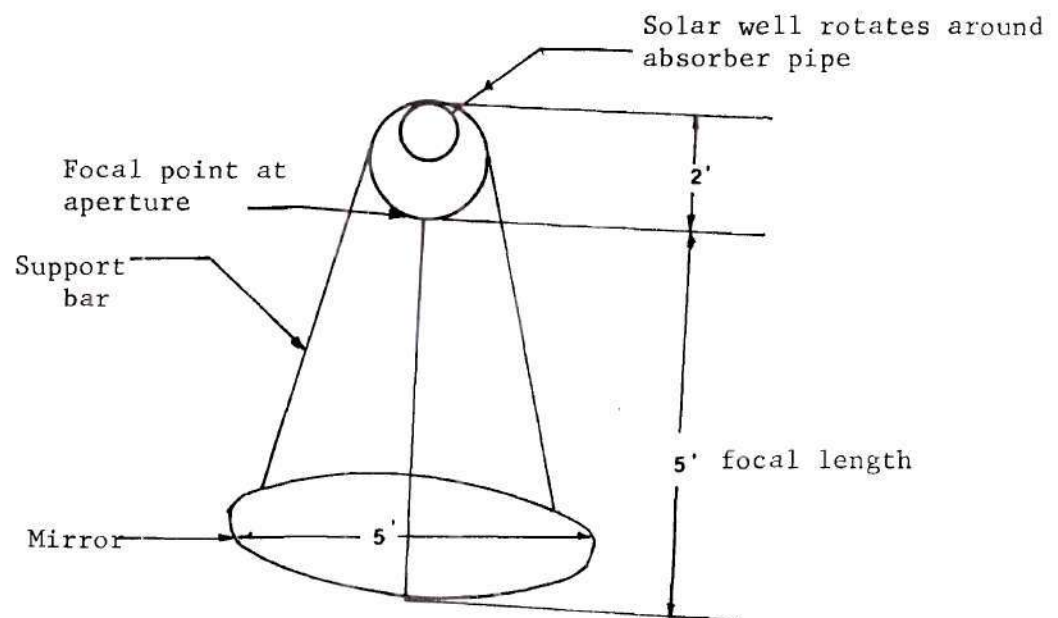


Figure 27. Alternate Tracking Mechanism Using the Solar Well of Figure 21, where Mirror and Solar Well Rotate Together (Requiring Seasonal Tilting of the Absorber Pipe to Follow the Sun's Latitude)

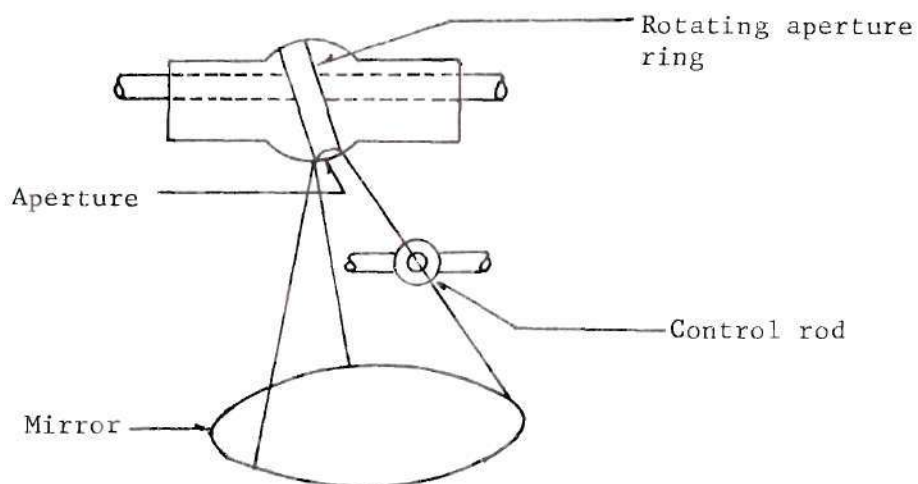


Figure 28. Rough Sketch of Horizontal Stationary Solar Well with Rotating Aperture Ring for Both Rotation and Season Change in Latitude

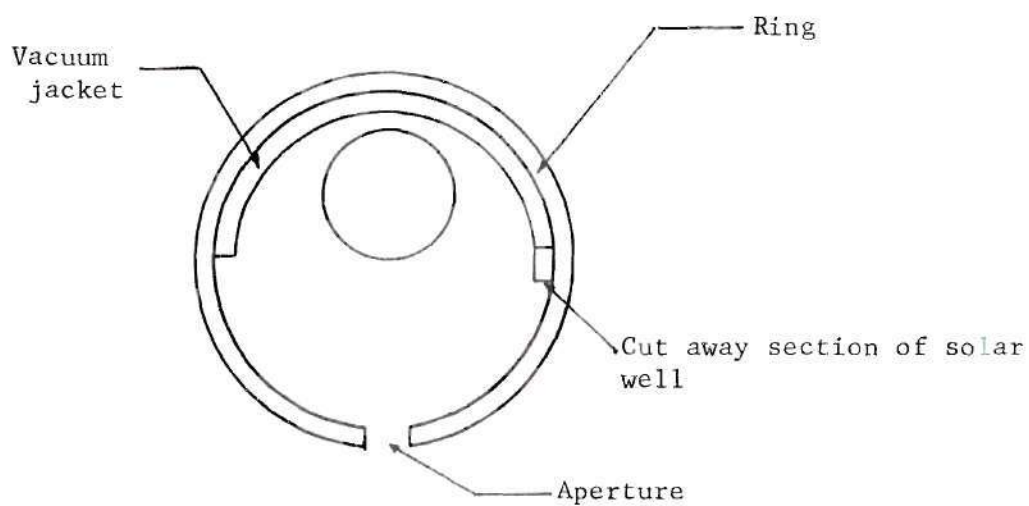


Figure 29. End View of Rotating Ring of Figure 28

## APPENDIX C

## PATENT SEARCH

An extensive patent search was made, and it revealed mostly patents discussing the solar distillation of water, solar heaters, solar furnaces, solar power plants, solar refrigeration utilizing solar stills, solar thermoelectric generator, etc. Only a few recent patents discussed intensive collectors; these were patents numbers 1479923, 2845724, 3058394, 3105486, 3334958, and there were also some much older patents. The Frensel lens patent (3334958) may be of help to get high concentration ratios at a reduced price of the collector system.

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